Mean-Variance Investing

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Abstract and Keywords
Mean-variance investing is all about diversification. By exploiting the interaction of assets with each other, so one asset’s gains can make up for another asset’s losses, diversification allows investors to increase expected returns while reducing risks. In practice, mean-variance portfolios that constrain the mean, volatility, and correlation inputs to reduce sampling error have performed much better than unconstrained portfolios. These constrained special cases include equal-weighted, minimum variance, and risk parity portfolios.

Keywords: diversification, mean-variance frontier, capital allocation line, mean-variance efficient, tangency portfolio, free lunch, socially responsible investing, risk parity, minimum variance, estimation risk, home bias, stock nonparticipation puzzle
Chapter Summary
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1. Norway and Wal-Mart
On June 6, 2006, the Norwegian Ministry of Finance announced that the Norwegian sovereign wealth fund, officially called “The Norwegian Government Pension Fund—Global,” had sold Wal-Mart Stores Inc. on the basis of “serious/systematic violations of human rights and labor rights.” As one of the largest funds in the world and a leader in ethical investing, Norway’s decision to exclude Wal-Mart was immediately noticed. Benson Whitney, the U.S. ambassador to Norway, complained that the decision was arbitrarily based on unreliable research and unfairly singled out an American company. Wal-Mart disputed Norway’s decision and sent two senior executives to plead its case before the Ministry of Finance.

Norway is one of the world’s largest oil exporters. Norway first found oil in the North Sea in 1969 and quickly found that its resulting revenue was distorting its economy. During the 1970s and 1980s, Norway experienced many symptoms of the Dutch disease (see chapter 1), with growing oil revenues contributing to a less competitive and shrinking manufacturing sector. When oil prices slumped in the mid-1980s, overreliance on oil revenue contributed to a period of slow economic growth. Sensibly, Norway decided to diversify.

Norway’s “Government Petroleum Fund” was set up in 1990 to channel some of the oil revenue into a long-term savings mechanism. The fund served two purposes: (i) it diversified oil wealth into a broader portfolio of international securities, improving Norway’s risk–return trade-off, and (ii) it inoculated Norway from the Dutch disease by quarantining wealth overseas, only gradually letting the oil money trickle into the
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economy. In January 2006, the fund was renamed “The Norwegian Government Pension—Global,” although it had no explicit pension liabilities. The new title conveyed the fund’s goal of managing its capital to meet long-term government obligations and to benefit future generations.

At first the fund was invested only in government bonds. In 1998 the investment universe was enlarged to allow a 40% allocation to equities, and that was raised to 60% in 2007. In 2010 the fund was permitted to invest up to 5% of assets in real estate, and Norway bought its first properties, in London and Paris, in 2011. While the asset universe of the fund had gradually broadened, since the fund’s inception the “reluctant billionaires of Norway” have always sought to meaningfully invest their fortune in line with the country’s social ethos. Norway practiced socially responsible investing (SRI). In 2005, government regulation was passed making this formal. The regulation stated that:

- The financial wealth must be managed so as to generate a sound return in the long term, which is contingent on sustainable development in the economic, environmental and social sense.

- The fund should not make investments that constitute an unacceptable risk that the fund may contribute to unethical acts or omissions, such as violations of human rights, gross corruption, or severe environmental damages.

The Ministry of Finance appointed an independent Council on Ethics, which issued recommendations on whether an investment constituted a violation of the fund’s ethical guidelines. If there was unacceptable risk, the Council would recommend the exclusion of a company. The Council continuously monitored all companies in the fund’s portfolio to uncover possible violations using publicly available information, media sources, national and international organizations, and independent experts.

In April 2005 the Council began examining alleged unethical activities by Wal-Mart. These included many reported violations of labor laws and human rights, including reports of child labor, serious violations of working hour regulations, paying wages below the legal minimum, hazardous working conditions, and unreasonable punishment. The Council found widespread gender discrimination. Wal-Mart stopped
workers from forming unions. There were reports of children performing dangerous work and the use of illegal immigrant labor.

In September 2005, the Council sent a letter to Wal-Mart asking the company to comment on the alleged human rights violations. Wal-Mart acknowledged the letter but did not otherwise respond.

From January until March 2006, the Ministry conducted its own assessment. The Ministry found that exercising the fund’s ownership rights through an activist approach would not be effective in influencing Wal-Mart’s business practices. Divestment decisions were always considered the last resort, but in Wal-Mart’s case the Ministry decided it was appropriate.

When the Ministry announced on June 6, 2006 that it had sold all its holdings in Wal-Mart, it quoted the report from the Council of Ethics:

What makes this case special is the sum total of ethical norm violations, both in the company’s own business operations and in the supplier chain. It appears to be a systematic and planned practice on the part of the company to hover at, or cross, the bounds of what are accepted norms for the work environment. Many of the violations are serious, most appear to be systematic, and altogether they form a picture of a company whose overall activity displays a lack of willingness to countervail violations of norms in its business operations.4

Excluding companies is not without cost: by shrinking its universe, Norway’s investment opportunities were smaller, and it lost diversification benefits and lowered its best risk-return trade-off. As more companies were excluded, there were further losses in diversification benefits. In January 2010, Norway excluded all tobacco companies. What did these exclusions do to the fund’s maximum attainable risk-return trade-off? How much did it cost Norway to be ethical?

In this chapter I cover mean-variance investing. This is by far the most common way to choose optimal portfolios. The main takeaway is that diversified portfolios should be selected because investors can reduce risk and increase returns. The
underlying concept of diversification can be implemented in different ways, and many of the approaches popular at the time of writing, like risk parity and minimum variance portfolios, are special cases of unconstrained mean-variance portfolios. An advantage of mean-variance investing is that it allows diversification benefits (and losses) to be measured in a simple way. We will later use mean-variance investing concepts to estimate how much Norway is losing in choosing to be socially responsible—in other words, to answer the question, how much does it cost Norway to divest Wal-Mart?

(p.74) 2. Mean-Variance Frontiers
Mean-variance frontiers depict the best set of portfolios that an investor can obtain (only considering means and volatilities, of course!). Let’s start by considering a U.S. investor contemplating investing only in U.S. or Japanese equities.

2.1. United States and Japan
In the 1980s Japan seemed poised to take over the world. Figure 3.1 plots cumulated returns of U.S. and Japanese equities from January 1970 to December 2011 using MSCI data. (I also use these data for the other figures involving G5 countries in this chapter.) Japanese returns are plotted in the solid line and U.S. returns are shown in the dashed line. Japanese equities skyrocketed in the 1980s. Many books were written on Japan’s stunning success, like Vogel’s (1979) *Japan as Number One: Lessons for America*. Flush with cash, Japanese companies went on foreign buying binges. Japanese businesses bought marquee foreign companies: Universal Studios and Columbia Records were sold to Matsushita Electric and Sony, respectively. The Japanese also bought foreign trophy real estate. The Mitsubishi Estate Company of Tokyo purchased Rockefeller Center in 1989. In 1990 the famous Pebble Beach golf course was sold to a Japanese businessman, Minuro Isutani. Figure 3.1 shows that the United States did well during the 1980s too, but not nearly as well as booming Japan.
Then everything crashed. Isutani had bought Pebble Beach a year after the Nikkei had hit its peak in 1989, and he was later investigated for money laundering by the FBI. Figure 3.1 shows that since 1990 Japanese stocks have been flat. But while Japan was languishing, the United States boomed. Even so, Japanese cumulated returns were higher at December 2011 than U.S. cumulated returns. Since 2000, Figure 3.1 shows that the United States and Japan have a greater tendency to move together. They jointly slowed during the early 2000s, experienced bull markets during the mid-2000s, and then crashed during the financial crisis of 2007–2008. Over the whole sample, however, Japan has moved very differently from the United States.

The average return and volatility for the United States in Figure 3.1 are 10.3% and 15.7%, respectively. The corresponding numbers for Japan are 11.1% and 21.7%, respectively. We plot these rewards (means) and risks (volatilities) in mean-standard deviation space in Figure 3.2. The United States is represented by the square, and Japan is represented by the circle. The x-axis is in standard deviation units, and the y-axis units are average returns.
The curve linking the United States and Japan in Figure 3.2 is the *mean-variance frontier*. Like the literature, I use the terms mean-variance frontier and mean-standard deviation frontier interchangeably as the two can be obtained simply by squaring, or taking a square root of, the x-axis depending on whether one (p.76) uses volatility or variance units. The mean-variance frontier for the United States and Japan represents all combinations of the United States and Japan. Naturally, the square representing the United States is a 100% U.S. portfolio, and the circle representing Japan is a 100% Japanese portfolio. All the other positions on the mean-variance frontier represent portfolios containing different amounts of the United States and Japan.

The mean-variance frontier is a *parabola*, or a bullet. The top half of the mean-variance frontier is *efficient*: an investor cannot obtain a higher reward, or expected return, for a given level of risk measured by volatility. Investors will choose portfolios on the top, efficient part of the frontier. The United States sits on the underbelly of the bullet. You can achieve a higher expected return for the same volatility by moving onto the top half of the frontier. The United States is an *inefficient* portfolio. *No one should hold a 100% U.S. portfolio.*

2.2. Diversification

In the *Merchant of Venice*, Shakespeare tells us why we should diversify:

... I thank my fortune for it,
My ventures are not in one bottom trusted,
Nor to one place; nor is my whole estate
Upon the fortune of this present year:
Therefore my merchandise makes me not sad.

This is spoken by the merchant Antonio, who diversifies so that all of his risk is not tied up in just one ship ("one bottom"). Similarly, we don’t want to bet everything on just the United States or Japan in isolation ("one place").

The fact that Japanese equities have moved differently from U.S. equities, especially over the 1980s and 1990s, causes the mean-variance frontier to bulge outwards to the left. The correlation between the United States and Japan is 35% over the sample. (The United States–Japan correlation post-2000 is 59%, still far below one.) Mean-variance frontiers are like the Happy (or Laughing) Buddha: the fatter the stomach or bullet, the more prosperous the investor becomes. Notice that the left-most point on the mean-variance frontier in Figure 3.2 has a lower volatility than either the United States or Japan. This portfolio, on the left-most tip of the bullet, is called the minimum variance portfolio.

Starting from a 100% U.S. portfolio (the square in Figure 3.2), an investor can improve her risk–return trade-off by including Japanese equities. This moves her position from the United States (the square) to Japan (the circle) and the investor moves upward along the frontier in a clockwise direction. Portfolios to the right-hand side of the circle (the 100% Japan position) represent levered portfolios. Portfolios on the top half of the frontier past the circle are constructed by shorting the United States, like a -30% position, and then investing the short proceeds in a levered Japanese position, which would be 130% in this case. All the efficient portfolios lying on the top half of the frontier—those portfolios with the highest returns for a given level of risk—contain Japanese equities. The minimum variance portfolio, which is the left-most tip of the mean-variance bullet, also includes Japan.

The American investor can improve her risk–return trade-off by holding some of Japan because Japan provides diversification benefits. This is the fundamental concept in mean-variance investing, and it corresponds to the common adage "don’t put all your eggs in one basket." The United States and Japan held together are better than the United States held alone. Owning both protects us from the
catastrophe that one individual investment will be lost. The advantages of diversification imply that we cannot consider assets in isolation; we need to think about how assets behave together. This is the most important takeaway of this chapter.

Diversified, efficient portfolios of the United States and Japan have higher returns and lower risk than the 100% U.S. position. Why? When the investor combines the United States and Japan, the portfolio reduces risk because when one asset does poorly, another might do well. It’s like buying insurance (except that the purchaser of insurance loses money, on average, whereas an investor practicing diversification makes more money, on average). When the United States does relatively poorly, like during the 1980s, Japan has a possibility of doing well. Some of the risk of the U.S. position is avoidable and can be offset by holding Japan as insurance.

What about the opposite? During the 1990s Japan was in the doldrums and the United States took off. Hindsight tells us that the U.S. investor would have been better off holding only the United States. Yes, he would—ex post. But forecasting is always hard. At the beginning of the 1990s, the investor would have been better off on an ex-ante basis by holding a portfolio of both the United States and Japan. What if the roles were reversed so that in the 1990s Japan did take over the world, and the U.S. swapped places with Japan? Holding Japan in 1990 diversified away some of this ex-ante risk.

The formal theory behind diversification was developed by Harry Markowitz (1952), who was awarded the Nobel Prize in 1990. The revolutionary capital asset pricing model (CAPM) is laid on the capstone of mean-variance investing, and we discuss that model in chapter 6. The CAPM pushes the diversification concept further and derives that an asset’s risk premium is related to the (lack of) diversification benefits of that asset. This turns out to be the asset’s beta.

\[(p.78)\] Mathematically, diversification benefits are measured by covariances or correlations. Denoting \( r_p \) as the portfolio return, the variance of the portfolio return is given by

\[
\text{var}(r_p) = w_{US}^2 \text{var}(r_{US}) + w_{JP}^2 \text{var}(r_{JP}) + 2w_{US}w_{JP} \text{cov}(r_{US}, r_{JP})
= w_{US}^2 \text{var}(r_{US}) + w_{JP}^2 \text{var}(r_{JP}) + 2w_{US}w_{JP} \rho_{US,JP} \sigma_{US} \sigma_{JP},
\]

\[(3.1)\]
where $r_{US}$ denotes U.S. returns, $r_{JP}$ denotes Japanese returns, and $w_{US}$ and $w_{JP}$ are the portfolio weights held in the United States and Japan, respectively. The portfolio weights can be negative, but they sum to one, $w_{US} + w_{JP} = 1$, as the portfolio weights total 100%. (This constraint is called an admissibility condition.) The covariance, $\text{cov}(r_{US}, r_{JP})$, in the first line of equation (3.1) can be equivalently expressed as the product of correlation between the United States and Japan ($\rho_{US,JP}$), and the volatilities of the United States and Japan ($\sigma_{US}$ and $\sigma_{JP}$, respectively), $\text{cov}(r_{US}, r_{JP}) = \rho_{US,JP} \sigma_{US} \sigma_{JP}$.

Large diversification benefits correspond to low correlations. Mathematically, the low correlation in equation (3.1) reduces the portfolio variance. Economically, the low correlation means that Japan is more likely to pay off when the United States does poorly and the insurance value of Japan increases. This allows the investor to lower her overall portfolio risk. The more Japan does not look like the United States, the greater the benefit of adding Japan to a portfolio of U.S. holdings. Mean-variance investors love adding investments that act differently from those that they currently hold. The more dissimilar, or the lower the correlation, the better.\(^7\)

Figure 3.3 plots the United States and Japan mean-variance frontier for different correlation values. The solid line is the frontier with the 35.4% correlation in data. The dashed line is drawn with 0% correlation, and the dotted line drops the correlation to -50%. As the correlation decreases, the tip of the mean-variance frontier pushes to the left—the bullet becomes more pointed. The lower correlation allows the investor to reduce risk as Japan provides even more diversification.
2.3. G5 Mean-Variance Frontiers

In Figures 3.4 and 3.5 we add the United Kingdom, France, and Germany.

First consider Figure 3.4, which plots the mean-variance frontier for the G3: the United States, Japan, and the United Kingdom. The G3 frontier is shown in the solid line. For comparison, the old United States-Japan frontier is in the dashed line. Two things have happened in moving from the G2 (United States and Japan) to the G3 (United States, Japan, and the United Kingdom):
1. The frontier has expanded. The Happy Buddha becomes much happier adding the United Kingdom. The pronounced outward shift of the wings of the mean-variance bullet means that an investor can obtain a much higher return for a given standard deviation. (There is also a leftward shift of the frontier, but this is imperceptible in the graph.) Starting at any point on the United States-Japan frontier, we can move upwards on an imaginary vertical line and obtain higher returns for the same level of risk. Adding the United Kingdom to our portfolio provides further diversification benefits because now there is an additional country that could have high returns when the United States is in a bear market while before there was only Japan. There is also a chance that both the old United States and Japan positions would do badly; adding the United Kingdom gives the portfolio a chance to offset some or all of those losses.

2. All individual assets lie inside the frontier. Individual assets are dominated: diversified portfolios on the frontier do better than assets held individually. Now all countries would never be held individually. Diversification removes asset-specific risk and reduces the overall risk of the portfolio.

In Figure 3.5, we add Germany and France. The G5 mean-variance frontier is in the solid line, and it is the fattest: the United States-Japan and the United States-Japan-United Kingdom frontiers lie inside the G5 frontier. There is still a benefit in adding Germany and France from the United States-Japan-United Kingdom, but it is not as big a benefit as adding the United Kingdom starting from the United States-Japan. That is, although Happy Buddha continues getting happier by adding countries, the rate at which he becomes happier decreases. There are decreasing marginal diversification benefits as we add assets. As we continue adding assets beyond the G5, the frontier will continue to expand but the added diversification benefits become smaller.
Figures 3.4 and 3.5 show that if we add an asset, the mean-variance frontier gets fatter. Conversely, if we remove an asset, the mean-variance frontier shrinks. Removing assets, as Norway did by divesting Wal-Mart, can only cause the maximum Sharpe ratio to (weakly) decrease. Thus, constraining the fund manager’s portfolio by throwing out Norway has decreased investment opportunities and lowered the maximum achievable risk-return trade-off. We will later compute how the mean-variance frontier shrinks as we remove assets.

The mathematical statement of the problem in Figures 3.4 and 3.5 is:

\[
\begin{align*}
\min_{w} & \quad \text{var}(\hat{r}) \\
\text{subject to} & \quad \hat{r} = \mu^* \text{ and } \sum_{i} w_i = 1,
\end{align*}
\]

where the portfolio weight for asset \( i \) is \( w_i \). We find the combination of portfolio weights, \( \{w_i\} \), that minimizes the portfolio variance subject to two constraints. The first is that the expected return on the portfolio is equal to a target return, \( \mu^* \). The second is that the portfolio must be a valid portfolio, which is the admissibility condition that we have seen earlier. Students of operations research will recognize equation (3.2) as a quadratic programming problem, and what makes mean-variance investing powerful (but alas, misused; see below) is that there are very fast algorithms for solving these types of problems.
Figure 3.6 shows how this works pictorially. Choosing a target return of $\mu^* = 10\%$, we find the portfolio with the lowest volatility (or variance). We plot this with an X. Then we change the target return to $\mu^* = 12\%$. Again we find the portfolio with the lowest volatility and plot this with another X. The mean-variance frontier is drawn by changing the target return, $\mu^*$ and then linking all the Xs for each target return. Thus, the mean-variance frontier is a locus of points, where each point denotes the minimum variance achievable for each expected return.
2.4. Constrained Mean-Variance Frontiers

So far, we have constructed unconstrained mean-variance frontiers. But investors often face constraints on what types of portfolios that they can hold. One constraint faced by many investors is the inability to short. When there is a no short-sales constraint, all the portfolio weights have to be positive (|\(w_i| \geq 0\)), and we can add this constraint to the optimization problem in equation (3.2).

Adding short-sale constraints changes the mean-variance frontier, sometimes dramatically. Figure 3.7 contrasts the mean-variance frontier where no shorting is permitted, in the solid line, with the unconstrained frontier, drawn in the dotted line. The constrained mean-variance frontier is much smaller than the unconstrained frontier and lies inside the unconstrained frontier. The constrained frontier is also not bullet shaped. Constraints inhibit what an investor can do and an investor can only be made (weakly) worse off. If an investor is lucky, the best risk–return trade-off is unaffected by adding constraints. We see this in Figure 3.7 in the region where the constrained and the unconstrained mean-variance frontiers lie on top of each other. But generally constraints cause an investor to achieve a worse risk–return trade-off. Nevertheless, even with constraints, the concept of diversification holds: the investor can reduce risk by holding a portfolio of assets rather than a single asset.

2.5. The Risks of Not Diversifying

Many people hold a lot of stock in their employer. Poterba (2003) reports that for large defined-contribution pension plans, the share of assets in own-company stock is around 40%. This comes usually, but not always, from discounted purchases and is designed by

Figure 3.7
companies to encourage employee loyalty. For individuals, such concentrated portfolios can be disastrous—as employees of Enron (bankrupt in 2001), Lucent (which spiraled downward after it was spun off from AT&T in 1996 and was bought for a pittance by Alcatel in 2006), and Lehman Brothers (bankrupt in 2008) found out. Enron employees had over 60% of their retirement assets in company stock when Enron failed. According to the Employee Benefits Research Institute, among 401(k) plan participants who had equity exposure in 2009, 12% had company stock as their only equity investment. For equity market participants in their sixties, 17% had equity exposure to equity markets only through their employers’ stock.

While the cost of not diversifying becomes painfully clear when your company goes bankrupt, mean-variance investing reveals that there is a loss even when your employer remains solvent. Individuals can generate a higher risk–return trade-off by moving to a diversified portfolio. Poterba (2003) computes the cost of not diversifying a retirement account relative to simply investing in the diversified S&P 500 portfolio. Assuming that half of an individual’s assets are invested in company stock, the certainty equivalent cost (see chapter 3) of this concentrated position is about 80% of the value of investing in the diversified S&P 500 portfolio. This is a substantial reduction in utility for investors. An individual should regularly cash out company stock, especially if the stock is rising faster than other assets in his portfolio. As own-company stock rises relative to other assets, it represents even greater concentrated risk for that investor. (Furthermore, your human capital itself is concentrated with that employer, see chapter 5.)

(p.84) The wealthy often fail to diversify enough. JP Morgan’s 2004 white paper, “Beating the Odds: Improving the 15% Probability of Staying Wealthy,” identified excessive concentration as the number-one reason the very wealthy lose their fortunes. The 15% probability in the study’s title comes from the fact that in the first Forbes list of the richest four hundred people in America, fewer than 15% were still on the list a generation later. While the Forbes 400 tracks the mega-rich, the wealthy below them are also likely to lose their wealth. Kennickell (2011) reports that of the American
households in the wealthiest 1% in 2007, approximately one-third fell out of the top 1% two years later.

Entrepreneurs and those generating wealth from a single business often find diversification counterintuitive. After all, wasn’t it concentrated positions that generated the wealth in the first place? This is the business that they know best, and their large investment in it may be illiquid and hard to diversify. But diversification removes company-specific risk that is outside the control of the manager. Over time, prime real estate ceases to be prime, and once-great companies fail because their products become obsolete. While some companies stumble due to regulatory risk, macro risk, technological change, and sovereign risk, other companies benefit. Diversification reduces these avoidable idiosyncratic risks. JP Morgan reports that of the five hundred companies in the S&P 500 index in 1990, only half remained in the index in 2000. Of the thirty titans comprising the original Dow Jones Industrial Average in 1896, only one remains: General Electric. This is testament to the need to diversify, diversify, and diversify if wealth is to be preserved.

Institutional investors also fail to sufficiently diversify. Jarrell and Dorkey (1993) recount the decline of the University of Rochester’s endowment. In 1971, Rochester’s endowment was $580 million, making it the fourth largest private university endowment in the country. In 1992, it ranked twentieth among private university endowments. What happened? From 1970 to 1992, the endowment earned only 7% compared to a typical 60%/40% equities–bonds portfolio return of 11%. Had Rochester simply invested in this benchmark portfolio, the endowment would have ranked tenth among private university endowments in 1992. By 2011, Rochester had dropped to thirtieth place. A big reason for Rochester’s underperformance was excessive concentration in local companies, especially Eastman Kodak, which filed for bankruptcy in 2012.

Boston University is another example. Over the 1980s and 1990s, Boston University invested heavily in Seragen Inc., then a privately held local biotech company. According to Lerner, Schoar, and Wang (2008), Boston University provided at least $107 million to Seragen from 1987 to 1997—a fortune considering the school’s endowment in 1987 was $142 million. Seragen went public but (p.85) suffered setbacks. In 1997,
the University’s stake was worth only $4 million. Seragen was eventually bought by Ligand Pharmaceuticals Inc. in 1998 for $30 million.

Norway’s sovereign wealth fund, in contrast, was created precisely to reap the gains from diversification. Through its sovereign wealth fund, Norway swaps a highly concentrated asset—oil—into a diversified financial portfolio and thus improves its risk-return trade-off.

2.6. Home Bias
Mean-variance investing prescribes that investors should never hold a 100% U.S. portfolio. Many investors do not take advantage of the benefits of international diversification and instead hold only domestic assets. This is the home bias puzzle.\(^{13}\)

One measure of home bias is the extent to which domestic investors’ equity holdings differ from the world equity market portfolio. The world market is a diversified portfolio and, according to the CAPM, which adds equilibrium conditions to the mean-investing framework (see chapter 6), it is the optimal portfolio which investors should hold. (I show below, in section 5, that the market portfolio performs well in a horse race with other mean-variance diversified portfolios.) Ahearne, Griever, and Warnock (2004) contrast the 12% proportion of foreign equities in U.S. investors’ portfolios with the approximate 50% share of foreign equities in the world market portfolio. Home bias is not just a U.S. phenomenon. Fidora, Fratzscher, and Thimann (2007) report the share of foreign assets in U.K. investors’ portfolios is 30% compared to the non-U.K. weight of 92% in the world market portfolio. Japan is even more home biased, with domestic investors owning fewer than 10% foreign equities while the non-Japanese positions in the world market portfolio are greater than 90%.

What accounts for home bias?
1. Correlations vary over time
The diversification benefits of international investing depend on there being low correlations (or at least correlations that are not near one in absolute value). If correlations of international equity returns increase when there are global crashes—such as in 1987 (stock market crash), 1998 (emerging market crisis brought about by Russia’s default), and the financial crisis over 2008–2009—then international diversification benefits may not be forthcoming when investors most desire them. Correlations of international stock markets do increase, and diversification benefits correspondingly decrease, during global bear markets. But Ang and Bekaert (2002) show that there (p.86) are large benefits in international investments despite correlations changing through time.14

2. Exchange rate risk
For a domestic investor, foreign equities bring exposure to international stock markets as well as foreign exchange rates.15 Exchange rate movements are an additional source of risk, which may cause domestic investors to shun foreign assets. This explanation was rejected in an early study by Cooper and Kaplanis (1994). More recently, Fidora, Fratzscher, and Thimann (2007) show that exchange rate volatility can explain some, but far from all, of the home bias puzzle.
3. Transaction costs
Transaction costs for investing abroad have fallen over the last thirty years, and the degree of home bias has also fallen over this time. Ahearne, Griever, and Warnock (2004) compute a home bias ratio, which is equal to one minus the ratio of the share of foreign equities held by U.S. residents to the share of foreign equities in the world portfolio. The home bias ratio is equal to zero when there is no home bias and equal to one when domestic investors are totally home biased. Ahearne, Griever, and Warnock show that the home bias ratio dropped from almost 1.0 in the early 1980s to below 0.8 in the early 2000s.

We should, however, be skeptical of transaction costs explanations. The lucky U.S. investor does not need to invest directly in foreign markets—foreign companies have long listed their shares on U.S. exchanges through American Depositary Receipts (ADRs) where transaction costs are low. And there are many financial intermediaries, like mutual funds, specializing in international assets, which U.S. investors can easily access. Viewed more broadly, transaction costs could also include the cost of becoming informed about the benefits of diversification. Both actual transaction costs and the costs of becoming financially literate are out of whack with the costs that economic models predict are necessary to match the pronounced home bias of investors. Glassman and Riddick (2001), for example, estimate that transactions costs need to be extraordinarily high—more than 12% per year—to explain observed home bias.
4. Asymmetric information
U.S. investors may be disadvantaged in foreign markets because foreigners know more about stocks in their local markets than U.S. investors do. Being less informed than foreigners, U.S. investors optimally choose to hold less foreign equity. One implication of this story is that domestic investors should do better than foreigners in local markets.


5. Behavioral biases
Huberman (2001) argues that “people simply prefer to invest in the familiar.” The lack of international diversification is then a result of people sticking to what they know. Morse and Shive (2011) assert that home bias is due to patriotism and find that countries whose residents are less patriotic exhibit less home bias.

The home bias puzzle literature is a good example of positive economics (see chapter 2). We still do not know, however, what prevents investors from taking up these opportunities. The concept of diversification from mean-variance investing is normative; sections 2.1 to 2.4 have demonstrated large benefits of international investing. Investors should seize the chance to diversify and improve their risk-return trade-offs. Your money should be like the testy adolescent who can’t wait to leave home: invest abroad and reap the benefits.

2.7. Is Diversification Really a Free Lunch?
Diversification has been called the only “free lunch” in finance and seems too good to be true. If you hold (optimized) diversified portfolios, you can attain better risk-return trade-offs than holding individual assets. Is it really a free lunch?
Yes, if you only care about portfolio means and variances.

Mean-variance investing, by definition, only considers means and variances. Portfolio variances are indeed reduced by holding diversified portfolios of imperfectly correlated assets (see equation (3.1)). In this context, there is a free lunch. (p. 88) But what if an investor cares about other things? In particular, what if the investor cares about downside risk and other higher moment measures of risk?

Variances always decrease when assets with nonperfect correlations are combined. This causes improvements in returns and reductions in risk in mean-variance space. But other measures of risk do not necessarily diminish when portfolios are formed. For example, a portfolio can be more negatively skewed and thus have greater downside risk than the downside risk of each individual asset.\(^{18}\) Most investors care about many more risk measures than simply variance.

Diversification is not necessarily a free lunch when other measures of risk are considered. Nevertheless, from the viewpoint of characterizing the tails of asset returns, standard deviation (or variance) is the most important measure. Furthermore, optimal asset weights for a general utility function can be considered to be mean-variance weights to a first approximation, but we may have to change an investor’s risk aversion in the approximation (see chapter 2).\(^{19}\) While variance is the first-order risk effect, in some cases the deviations from the mean-variance approximation can be large. You still need to watch the downside.

In the opposite direction, diversification kills your chances of the big lottery payoff. If you are risk seeking and want to bet on a stock having a lucky break—and hope to become a billionaire from investing everything in the next Microsoft or Google—diversification is not for you. Since diversification reduces idiosyncratic risk, it also limits the extremely high payoffs that can occur from highly concentrated positions. The risk-averse investor likes this because it also limits the catastrophic losses that can result from failing to diversify. Just ask the employees of Enron and Lehman and the faculty and students at the University of Rochester and Boston University.
The overall message from mean-variance investing is that diversification is good. It minimizes risks that are avoidable and idiosyncratic. It views assets holistically, emphasizing how they interact with each other. By diversifying, investors improve their Sharpe ratios and can hold strictly better portfolios—portfolios that have higher returns per unit of risk or lower risk for a given target return—than assets held in isolation.

3. Mean-Variance Optimization

We’ve described the mean-variance frontier and know that the best investment opportunities lie along it. Which of these efficient portfolios on the mean-variance frontier should we pick?

(p.89) That depends on each investor’s risk aversion. We saw in the previous chapter that we can summarize mean-variance preferences by indifference curves. Maximizing mean-variance utility is equivalent to choosing the highest possible indifference curve. Indifference curves correspond to the individual maximizing mean-variance utility (see equation (2.4) in chapter 2 restated here):

\[
\max_{\{w\}} \frac{1}{2} \text{var}(r_p), \quad \text{subject to any constraints.}
\]

The coefficient of risk aversion, \( g \), is specific to each individual. The weights, \( \{w\} \), correspond to the risky assets in the investor’s universe. Investment in the risk-free asset, if it is available, constitutes the remaining investment (all the asset weights, including the risk-free asset position, sum to one).

3.1. Without a Risk-Free Asset

Figure 3.8 shows the solution method for the case without a risk-free asset.\(^{20}\) The left graph in the top row shows the indifference curves. As covered in chapter 2, one particular indifference curve represents one level of utility. The investor has (p.90) the same utility for all the portfolios on a given indifference curve. The investor moves to higher utility by moving to successively higher indifference curves. The right graph in the top row is the mean-variance frontier that we constructed from section 2. The frontier is a property of the asset universe, while the indifference curves are functions of the risk aversion of the investor.
We bring the indifference curves and the frontier together in the bottom row of Figure 3.8. We need to find the tangency point between the highest possible indifference curve and the mean-variance frontier. This is marked with the $X$. Indifference curves lying above this point represent higher utilities, but these are not attainable—we must lie on the frontier. Indifference curves lying below the $X$ represent portfolios that are attainable as they intersect the frontier. We can, however, improve our utility by shifting to a higher indifference curve. The highest possible utility achievable is the tangency point $X$ of the highest possible indifference curve and the frontier.

Let’s go back to our G5 countries and take a mean-variance investor with a risk aversion coefficient of $\gamma = 3$. In Figure 3.9, I plot the constrained (no shorting) and unconstrained mean-variance frontiers constructed using U.S., Japanese, U.K., German, and French equities. The indifference curve corresponding to the maximum achievable utility is drawn and is tangent to the frontiers at the asterisk. At this point, both the constrained and unconstrained frontiers overlap. The optimal portfolio at the tangency point is given by:
### Optimal Portfolio

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>JP</th>
<th>U.K.</th>
<th>GR</th>
<th>FR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Portfolio</td>
<td>0.45</td>
<td>0.24</td>
<td>0.16</td>
<td>0.11</td>
<td>0.04</td>
</tr>
</tbody>
</table>
This portfolio is heavily weighted towards the United States and Japan with weights of 45% and 24%, respectively. Note that by construction, this portfolio consists only of risky assets, so the portfolio weights sum to one. (The weights in this example are also fairly close to the market capitalization weights of these countries.) With a risk-free rate of 1%, the Sharpe ratio corresponding to this optimal portfolio is 0.669.

3.2. With a Risk-Free Asset

The addition of a risk-free asset expands the investor's opportunities considerably. Since there is only one period, the risk-free asset has no variance. Think of T-bills as an example of a security with a risk-free return. (There is some small default risk in T-bills, which you should ignore for now; I cover sovereign default risk in chapter 14.)

When there is a risk-free asset, the investor proceeds in two steps:

1. Find the best risky asset portfolio. This is called the mean-variance efficient (MVE) portfolio, or tangency portfolio, and is the portfolio of risky assets that maximizes the Sharpe ratio.21
2. Mix the best risky asset portfolio with the risk-free asset. This changes the efficient set from the frontier into a wider range of opportunities. The efficient set becomes a capital allocation line (CAL), as I explain below.

The procedure of first finding the best risky asset portfolio (the MVE) and then mixing it with the risk-free asset is called two-fund separation. It was originally developed by James Tobin (1958), who won the Nobel Prize in 1981. Given the
limited computing power at the time, it was a huge breakthrough in optimal portfolio choice.

Let’s first find the best risky asset portfolio, or MVE. Assume the risk-free rate is 1%. Figure 3.10 plots our now familiar mean-variance frontier for the G5 and marks the MVE with an asterisk. The dashed diagonal line that goes through the MVE is the CAL. (We encountered the CAL in the previous chapter.) The CAL starts at the risk-free rate, which is 1% in Figure 3.10 and is tangent to the mean-variance frontier. The tangency point is the MVE. The CAL is obtained by taking all combinations of the MVE with the 1% yielding risk-free asset. The MVE itself corresponds to a 100% position in only G5 equities and the intersection point of the CAL on the y-axis at 1% corresponds to a 100% risk-free position.

The slope of the CAL represents the portfolio’s Sharpe ratio. Since the CAL is tangent at the MVE, it represents the maximum Sharpe ratio that can be obtained by the investor. A line that starts from the risk-free rate of 1% on the y-axis but with a larger angle, which tilts closer to the y-axis, cannot be implemented as it does not intersect the frontier. The frontier represents the set of best possible portfolios of G5 risky assets, and we must lie on the frontier. A line that starts from the risk-free rate of 1% on the y-axis but with a lesser angle than the CAL, which tilts closer to the x-axis, intersects the frontier. These are CALs that can be obtained in actual portfolios but do not represent the highest possible Sharpe ratio. The maximum Sharpe ratio is the tangency point, or MVE.
The MVE in Figure 3.10 has a Sharpe ratio of 0.671. It consists of:
## MVE Portfolio

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>JP</th>
<th>U.K.</th>
<th>GR</th>
<th>FR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVE Portfolio</td>
<td>0.53</td>
<td>0.24</td>
<td>0.12</td>
<td>0.10</td>
<td>0.02</td>
</tr>
</tbody>
</table>
All the portfolios that lie on the CAL have the same Sharpe ratio, except for the 100% risk-free position that corresponds to the risk-free rate of 1% on the y-axis.

Now that we've found the best risky MVE portfolio, the investor mixes the risk-free asset with the MVE portfolio. This takes us off the mean-variance frontier. Finding the optimal combination of the MVE with the risk-free asset is equivalent to finding the point at which the highest possible indifference curve touches the CAL. The tangency point is the investor's optimal portfolio. In Figure 3.11, we graph the CAL and show the optimal holding in the triangle for an investor with a risk aversion of $\gamma = 3$. The indifference curve that is tangent to this point—which corresponds to the maximum utility for this investor—is also plotted.

In Figure 3.11, the tangency point of the highest indifference curve and the CAL lies to the right of the MVE. This means the investor shorts the risk-free asset, or borrows money at 1%, and has a levered position in the MVE. The optimal positions corresponding to the triangle, which is the tangency MVE point, are:
Mean-Variance Investing

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>MVE Portfolio</td>
<td>0.80</td>
<td>0.37</td>
<td>0.18</td>
<td>0.15</td>
<td>0.03</td>
<td>-0.52</td>
</tr>
</tbody>
</table>
The proportions of the risky assets relative to each other in this optimal portfolio are the same as the weights of the MVE. That is, the 0.53 MVE weight of the United States is the same as $0.80/(0.80+0.37+0.18+0.15+0.03)$. The optimal position for the $\gamma=3$ investor has a Sharpe ratio of 0.671, which is the same as the CAL as it lies on the CAL.

How much has the investor gained in moving from our previous constrained setting in section 3.1 (no risk-free asset available) to the example with the risk-free asset included? The certainty equivalent of the tangency position with the (p. 94) short position in the risk-free rate is 0.085. The corresponding certainty equivalent restricting the investor to only risky asset positions obtained earlier is 0.077. Letting the investor have access to the (short) risk-free asset position represents a significant risk-free utility increase of eighty basis points.

### 3.3. Non-Participation in the Stock Market

Mean-variance investing predicts that with just equities and a risk-free asset, all investors should invest in the stock market except those who are infinitely risk averse. In reality, only half of investors put money in the stock market. This is the non-participation puzzle.

Table 3.12 reports equity market participation rates by households in the United States calculated by Li (2009) using data from the Panel Study of Income Dynamics (PSID), a household survey conducted by the University of Michigan, and the Survey of Consumer Finances (SCF) conducted by the Federal Reserve Board. (Obviously you can’t be poor if you have some savings, and researchers go further and exclude those with meager savings.)
### Table 3.12

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PSID</td>
<td>27%</td>
<td>31%</td>
<td>37%</td>
<td>28%</td>
<td>32%</td>
<td>29%</td>
<td>27%</td>
</tr>
<tr>
<td>SCF (excluding pensions and IRAs)</td>
<td>21%</td>
<td>23%</td>
<td>30%</td>
<td>32%</td>
<td>30%</td>
<td>29%</td>
<td>29%</td>
</tr>
<tr>
<td>SCF (including pensions and IRAs)</td>
<td>-</td>
<td>32%</td>
<td>39%</td>
<td>50%</td>
<td>52%</td>
<td>51%</td>
<td>51%</td>
</tr>
</tbody>
</table>
In Table 3.12, the PSID and SCF stock holdings track each other closely and have hovered around 30%. The SCF also counts stocks included in pension plans and IRAs and when these retirement assets are included, the proportion of households holding stocks increases to around 50%. There has been a general increase in stock market participation when retirement assets are included from around 30% in the 1980s to 50% in 2005. Yet about half of U.S. households do not hold any equities. This is not just an American phenomenon; Laakso (2010) finds that stock market participation in Germany and France is well below 50% including both direct investments and those made indirectly through mutual funds and investment accounts. Italy, Greece, and Spain have stock participation rates at approximately 10% or less.

Several explanations have been proposed for the high non-participation in stocks markets. Among these are:

1. **Investors do not have mean-variance utility.** We covered many more realistic utility functions in chapter 2. Utility functions that can capture the greater risk aversion investors have to downside losses can dramatically lower optimal holdings of equities. Investors with disappointment utility, in particular, will optimally not participate in the stock market, as shown by Ang, Bekaert, and Liu (2005).

2. **Participation costs.** These costs include both the transaction costs of actually purchasing equities, but more broadly they include the expense of becoming financially educated and the “psychic” cost of overcoming fears of investing in the stock market. Consistent with a participation cost explanation, Table 3.12 shows that more people have invested in stocks as equities have become easier to trade since the 1980s with the arrival of online trading and easier access to mutual funds. According to Vissing-Jørgensen (2002), a cost of just $50 in year 2000 prices explains why half of non-stockholders do not hold equities. Conversely, Andersen and Nielsen (2010) conclude that participation costs cannot be an explanation. In their somewhat morbid paper, they examine households inheriting stocks due to sudden deaths. These households pay no participation costs to
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enter stock markets, yet most of these households simply sell the entire equity portfolio and move the proceeds into risk-free bonds.

Several social factors are highly correlated with holding equities. Whether you invest in equities depends on whether your neighbor invests in equities and whether you are politically active. Non-equity holders may have less trust in markets than their peers. Investors’ expectations of returns are highly dependent on whether they have been burned by previous forays into the stock market. Provocatively, Grinblatt, Keloharju, and Linnainmaa (2012) find that the more intelligent the investor, the more he or she invests in stocks.

Whatever the reason, my advice is: Don’t be a non-participant. Invest in the stock market. You’ll reap the equity risk premium too (see chapter 8). But do so as part of a diversified portfolio.

4. Garbage In, Garbage Out
Mean-variance frontiers are highly sensitive to estimates of means, volatilities, and correlations. Very small changes to these inputs result in extremely (p.96) different portfolios. These problems have caused mean-variance optimization to be widely derided. The lack of robustness of “optimized” mean-variance portfolios is certainly problematic, but it should not take away from the main message of mean-variance investing that diversified portfolios are better than individual assets. How to find an optimal portfolio mean-variance portfolio, however, is an important question given these difficulties.

4.1. Sensitivity to Inputs
Figure 3.13 showcases this problem. It plots the original mean-variance frontier estimated from January 1970 to December 2011 in the solid line. The mean of U.S. equity returns in this sample is 10.3%. Suppose we change the mean to 13.0%. This choice is well within two standard error bounds of the data estimate of the U.S. mean. The new mean-variance frontier is drawn in the dashed line. There is a large difference between the two.
The mean-variance frontier portfolios corresponding to a target return of 12% in the original case (U.S. mean is 10.3%) and the new case (U.S. mean is 13.0%) are:

<table>
<thead>
<tr>
<th></th>
<th>U.S. mean = 10.3%</th>
<th>U.S. mean = 13.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S.</td>
<td>-0.0946</td>
<td>0.4101</td>
</tr>
<tr>
<td>JP</td>
<td>0.2122</td>
<td>0.3941</td>
</tr>
<tr>
<td>U.K.</td>
<td>0.4768</td>
<td>0.0505</td>
</tr>
<tr>
<td>GR</td>
<td>0.1800</td>
<td>0.1956</td>
</tr>
<tr>
<td>FR</td>
<td>0.2257</td>
<td>-0.0502</td>
</tr>
</tbody>
</table>

Previously, we didn’t have negative weights in the United States because we worked with the (constrained) optimal portfolio for a risk aversion of $\gamma = 3$. This corresponds to a target return of 11.0%. The portfolio on the frontier corresponding to a target return of 12% involves a short U.S. position of -9%. This small change in the target return and the resulting large change in the portfolio weights itself showcases the lack of robustness of mean-variance optimization.

Changing the U.S. mean to 13.0% has caused the U.S. position to change from -9% to 41%, the U.K. position to move from 48% to approximately 5%, and the French position to shrink from 23% to -5%. These are very large changes resulting from
a small change in the U.S. mean. No wonder Michaud (1989) calls mean-variance portfolios “error maximizing portfolios.”

4.2. What to Do?

Change Utility
My first recommendation is not to use mean-variance utility. Investors are fearful of other risks, they care about relative performance, like catching-up-with-the-Joneses or habit utility, and they dread losses much more than they cherish (p.97) gains. (For more general utility functions, see chapter 2.) Unfortunately, we have a dearth of commercial optimizers (none that I know about at the time of writing) that can spit out optimal portfolios for more realistic utility functions, but there are plenty of very fancy mean-variance optimizers. If you insist on (or are forced to use) mean-variance utility. . . .

Use Constraints
Jagannathan and Ma (2003) show that imposing constraints helps a lot. Indeed, raw mean-variance weights are so unstable that practitioners using mean-variance optimization always impose constraints. Constraints help because they bring back unconstrained portfolio weights to economically reasonable positions. Thus, they can be interpreted as a type of robust estimator that shrinks unconstrained weights back to reasonable values. We can do this more generally if we. . . .

Use Robust Statistics
Investors can significantly improve estimates of inputs by using robust statistical estimators. One class of estimators is Bayesian shrinkage methods.24 These estimators take care of outliers and extreme values that play havoc with traditional classical estimators. They shrink estimates back to a prior, or model, which is based on intuition or economics. For example, the raw mean estimated in a (p.98) sample would not be used, but the raw mean would be adjusted to the mean implied by the CAPM (see chapter 6), a multifactor model, or some value computed from fundamental analysis. Covariances can also be shrunk back to a prior where each stock in an industry, say, has the same volatility and correlation—which is reasonable if we view each stock in a given industry as similar to the others.25

No statistical method, however, can help you if your data are lousy.
Don’t Just Use Historical Data

Investors must use past data to estimate inputs for optimization problems. But many investors simply take historical averages on short, rolling samples. This is the worst thing you can do.

In drawing all of the mean-variance frontiers for the G5, or various subsets of countries, I used historical data. I plead guilty. I did, however, use a fairly long sample, from January 1970 to December 2011. Nevertheless, even this approximately forty-year sample is relatively short. You should view the figures in this chapter as what has transpired over the last forty years and not as pictures of what will happen in the future. As the investment companies like to say in small print, past performance is no guarantee of future returns. The inputs required for mean-variance investing—expected returns, volatilities, and correlations—are statements about what we think will happen in the future.

Using short data samples to produce estimates for mean-variance inputs is very dangerous. It leads to pro-cyclicality. When past returns have been high, current prices are high. But current prices are high because future returns tend to be low. While predictability in general is very weak, chapter 8 provides evidence that there is some. Thus, using a past data sample to estimate a mean produces a high estimate right when future returns are likely to be low. These problems are compounded when more recent data are weighted more heavily, which occurs in techniques like exponential smoothing.

An investor using a sample where returns are stable, like the mid-2000s right before the financial crisis, would produce volatility estimates that are low. But these times of low volatilities (and high prices) are actually periods when risk is high. Sir Andrew Crockett of the Bank of England says (with my italics):26 “The received wisdom is that risk increases in the recessions and falls in booms. In contrast, it may be more helpful to think of risk as *increasing* during upswings, as financial imbalances build up, and *materializing* in recessions.” The low estimates (p.99) of volatilities computed using short samples ending in 2007 totally missed the explosions in risks that materialized in the 2008–2009 financial crisis.

Use Economic Models
I believe that asset allocation is fundamentally a valuation problem. The main problem with using purely historical data, even with the profession’s best econometric toolkit, is that it usually ignores economic value. Why would you buy more of something if it is too expensive?

Valuation requires an economic framework. Economic models could also be combined with statistical techniques. This is the approach of Black and Litterman (1991), which is popular because it delivers estimates of expected returns that are “reasonable” in many situations. Black and Litterman start with the fact that we observe market capitalizations, or market weights. The market is a mean-variance portfolio implied by the CAPM equilibrium theory (see chapter 6). Market weights, which reflect market prices, embody the market’s expectations of future returns. Black and Litterman use a simple model—the CAPM—to reverse engineer the future expected returns (which are unobservable) from market capitalizations (which are observable). In addition, their method also allows investors to adjust these market-based weights to their own beliefs using a shrinkage estimator. I will use Black–Litterman in some examples below, in section 6.

An alternative framework for estimating inputs is to work down to the underlying determinants of value. In Part II of this book, I will build a case for thinking about the underlying factors that drive the risk and returns of assets. Understanding how the factors influence returns and finding which factor exposures are right for different investors in the long run enables us to construct more robust portfolios.

The concept of factor investing (see chapter 14), where we look through asset class labels to the underlying factor risks, is especially important in maximizing the benefits of diversification. Simply giving a group of investment vehicles a label, like “private equity” or “hedge funds,” does not make them asset classes. The naïve approach to mean-variance investing treats these as separate asset classes and plugs them straight into a mean-variance optimizer. Factor investing recognizes that private equity and hedge funds have many of the same factor risks as traditional asset classes. Diversification benefits can be overstated, as many investors discovered in 2008 when risky asset classes came crashing
down together, if investors do not look at the underlying factor risks.

**Keep It Simple (Stupid)**
The simple things always work best. The main principle of mean-variance investing is to hold diversified portfolios. There are many simple diversified portfolios, and they tend to work much better than the optimized portfolios computed in the full glory of mean-variance quadratic programming in equations (3.2) and (3.3). (p.100) Simple portfolios also provide strong benchmarks to measure the value-added of more complicated statistical and economic models.

The simplest strategy—an equally weighted portfolio—turns out to be one of the best performers, as we shall now see.

5. Special Mean-Variance Portfolios
In this section I run a horse race between several portfolio strategies, each of which is a special case of the full mean-variance strategy. Diversification is common to all the strategies, but they build a diversified portfolio in different ways. This leads to very different performance.

5.1. Horserace
I take four asset classes—U.S. government bonds (Barcap U.S. Treasury), U.S. corporate bonds (Barcap U.S. Credit), U.S. stocks (S&P 500), and international stocks (MSCI EAFE)—and track the performance of various portfolios from January 1978 to December 2011. The data are sampled monthly. The strategies implemented at time \( t \) are estimated using data over the past five years, \( t-60 \) to \( t \). The first portfolios are formed at the end of January 1978 using data from January 1973 to January 1978. The portfolios are held for one month, and then new portfolios are formed at the end of the month. I use one-month T-bills as the risk-free rate. In constructing the portfolios, I restrict shorting down to \(-100\%\) on each asset class.

Using short, rolling samples opens me up to the criticisms of the previous section. I do this deliberately because it highlights some of the pitfalls of (fairly) unconstrained mean-variance approaches. Consequently, it allows us to understand why some special cases of mean-variance perform well and others badly.
I run a horserace between:

**Mean-Variance Weights** where the weights are chosen to maximize the Sharpe ratio.

**Market Weights**, which are given by market capitalizations of each index.

**Diversity Weights**, which are (power) transformations of market weights recommended by Fernholz, Garvy, and Hannon (1998).

**Equal Weights**, or the 1/N rule, which simply holds one-quarter in each asset class. Duchin and Levy (2009) call this strategy the “Talmudic rule” since the Babylonian Talmud recommended this strategy approximately 1,500 years ago: “A man should always place his money, one third in land, a third in merchandise, and keep a third in hand.”

**Risk Parity** is the strategy du jour and chooses asset weights proportional to the inverse of variance [Risk Parity (Variance)] or to the inverse of volatility [Risk Parity (Volatility)]. The term “risk parity” was originally coined by Edward Qian in 2005.\(^{27}\) It has shot to prominence in the practitioner community because of the huge success of Bridgewater Associates, a large hedge fund with a corporate culture that has been likened to a cult.\(^ {28}\) Bridgewater launched the first investment product based on risk parity called the “All Weather” fund in 1996. In 2011 the founder of Bridgewater, Ray Dalio, earned $3.9 billion.\(^ {29}\) (I cover hedge funds in chapter 17). Bridgewater’s success has inspired many copycats. The original implementations of risk parity were done on variances, but there are fans of weighting on volatilities.\(^ {30}\)

**Minimum Variance** is the portfolio on the left-most tip of the mean-variance frontier, which we’ve seen before.

**Equal Risk Contributions** form weights in each asset position such that they contribute equally to the total portfolio variance.\(^ {31}\)

**Kelly** (1956) **Rule** is a portfolio strategy that maximizes the expected log return. In the very long run, it will maximize wealth. (I explain more in chapter 4.)
**Proportional to Sharpe Ratio** is a strategy that holds larger positions in assets that have larger realized Sharpe ratios over the last five years.

Over this sample a 100% investment in U.S. equities had a Sharpe ratio of 0.35. This is dominated by all the diversified portfolios in the horserace, except for the most unconstrained mean-variance portfolio. This is consistent with the advice from the example in section 2 where no one should hold a 100% U.S. equity portfolio; diversification produces superior portfolios with lower risk and higher returns.

Table 3.14 reports the results of the horserace. Mean-variance weights perform horribly. The strategy produces a Sharpe ratio of just 0.07, and it is trounced by all the other strategies. Holding market weights does much better, with a Sharpe ratio of 0.41. This completely passive strategy outperforms the Equal Risk Contributions and the Proportional to Sharpe Ratio portfolios (with Sharpe ratios of 0.32 and 0.45, respectively). Diversity Weights tilt the portfolio toward the asset classes with smaller market caps, and this produces better results than market weights.
### Table 3.14


<table>
<thead>
<tr>
<th>Strategy</th>
<th>Raw Return</th>
<th>Volatility</th>
<th>Sharpe Ratio</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean-Variance Weights</td>
<td>6.06</td>
<td>11.59</td>
<td>0.07</td>
<td>Maximizes Sharpe ratio</td>
</tr>
<tr>
<td>Market Weights</td>
<td>10.25</td>
<td>12.08</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td>Diversity Weights</td>
<td>10.14</td>
<td>10.48</td>
<td>0.46</td>
<td>Uses a transformation of market weights</td>
</tr>
<tr>
<td>Equal Weights (1/4)</td>
<td>10.00</td>
<td>8.66</td>
<td>0.54</td>
<td></td>
</tr>
<tr>
<td>Risk Parity (Variance)</td>
<td>8.76</td>
<td>5.86</td>
<td>0.59</td>
<td>Weights inversely proportional to variance</td>
</tr>
<tr>
<td>Risk Parity (Volatility)</td>
<td>9.39</td>
<td>6.27</td>
<td>0.65</td>
<td>Weights inversely proportional to volatility</td>
</tr>
<tr>
<td>Minimum Variance</td>
<td>7.96</td>
<td>5.12</td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td>Equal Risk Contributions</td>
<td>7.68</td>
<td>7.45</td>
<td>0.32</td>
<td>Equal contribution to portfolio variance</td>
</tr>
<tr>
<td>Kelly Rule</td>
<td>7.97</td>
<td>4.98</td>
<td>0.54</td>
<td>Maximizes expected log return</td>
</tr>
<tr>
<td>Proportional to Sharpe Ratio</td>
<td>9.80</td>
<td>9.96</td>
<td>0.45</td>
<td></td>
</tr>
</tbody>
</table>

**Average Asset Weights**

<table>
<thead>
<tr>
<th></th>
<th>Raw Return</th>
<th>Volatility</th>
<th>Sharpe Ratio</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. Govt Bonds</td>
<td>U.S. Corp Bonds</td>
<td>U.S. Stocks</td>
<td>International Stocks</td>
</tr>
<tr>
<td>Mean-Variance Weights</td>
<td>0.74</td>
<td>-0.05</td>
<td>0.06</td>
<td>0.25</td>
</tr>
<tr>
<td>Market Weights</td>
<td>0.14</td>
<td>0.08</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>Diversity Weights</td>
<td>0.19</td>
<td>0.15</td>
<td>0.33</td>
<td>0.32</td>
</tr>
<tr>
<td>Equal Weights (1/4)</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Average Asset Weights</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>U.S. Govt Bonds</td>
<td>U.S. Corp Bonds</td>
<td>U.S. Stocks</td>
<td>International Stocks</td>
</tr>
<tr>
<td>Risk Parity (Variance)</td>
<td>0.51</td>
<td>0.36</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>Risk Parity (Volatility)</td>
<td>0.97</td>
<td>-0.30</td>
<td>0.17</td>
<td>0.16</td>
</tr>
<tr>
<td>Minimum Variance</td>
<td>1.41</td>
<td>-0.51</td>
<td>0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>Equal Risk Contributions</td>
<td>0.50</td>
<td>0.42</td>
<td>0.25</td>
<td>-0.17</td>
</tr>
<tr>
<td>Kelly Rule</td>
<td>1.18</td>
<td>-0.29</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>Proportional to Sharpe Ratio</td>
<td>0.24</td>
<td>0.21</td>
<td>0.21</td>
<td>0.35</td>
</tr>
</tbody>
</table>
The simple Equal Weight strategy does very well with a Sharpe ratio of 0.54. What a contrast with this strategy versus the complex mean-variance portfolio (with a Sharpe ratio of 0.07)! The Equal Weight strategy also outperforms the market portfolio (with a Sharpe ratio of 0.41). De Miguel, Garlappi, and Uppal (2009) find that the simple $1/N$ rule outperforms a large number of other implementations of mean-variance portfolios, including portfolios constructed using robust Bayesian estimators, portfolio constraints, and optimal combinations of portfolios which I covered in section 4.2. The $1/N$ portfolio also produces a higher Sharpe ratio than each individual asset class position. (U.S. bonds had the highest Sharpe ratio of 0.47 in the sample.)

Risk Parity does even better than $1/N$. The outperformance, however, of the plain-vanilla Risk Parity (Variance) versus Equal Weights is small. Risk Parity (Variance) has a Sharpe ratio of 0.59 compared to the 0.54 Sharpe ratio for Equal Weights. Risk Parity based on volatility does even better and has the highest out-of-sample Sharpe ratio of all the strategies considered, at 0.65. When risk parity strategies are implemented on more asset classes (or factor strategies, see chapter 7) in practice, historical Sharpe ratios for risk parity strategies have often exceeded one.

The outperformance of the Minimum Variance portfolio versus standard mean-variance weights and the market portfolio has been known for at least twenty years. One reason that minimum variance portfolios outperform the market is that there is a tendency for low volatility assets to have higher returns than high volatility assets, which I cover in chapter 10, and the minimum variance portfolio overweights low volatility stocks. The last two strategies in Table 3.14 are the Kelly Rule and the Proportional to Sharpe Ratio strategies. Both also outperform the Mean-Variance Weights and the market portfolio in terms of Sharpe ratios. You would have been better off, however, using the simple $1/N$ strategies in both cases.

Figure 3.15 plots cumulated returns of the Market Weights, Equal Weights, Risk Parity (Variance), and Mean-Variance strategies. All these returns are scaled to have the same volatility as the passive market weight strategy. The dominance of the Equal Weights and Risk Parity strategies is
obvious. In addition, Figure 3.15 shows that the Risk Parity strategy has the smallest drawdown movements of the four strategies.

5.2. Why Does Unrestricted Mean-Variance Perform So Badly?

The optimal mean-variance portfolio is a complex function of estimated means, volatilities, and correlations of asset returns. There are many parameters to estimate. Optimized mean-variance portfolios can blow up when there are tiny errors in any of these inputs. In the horserace with four asset classes, there are just fourteen parameters to estimate and even with such a low number mean-variance does (p.105) badly. With one hundred assets, there are 5,510 parameters to estimate. For five thousand stocks (approximately the number of common stocks listed in U.S. markets) the number of parameters to estimate exceeds 12,000. The potential for errors is enormous.

Let’s view what happens when we move from the optimal mean-variance strategy and turn off some of the inputs, so that we are relieved from estimating means, volatilities, correlations, or combinations of all three. Table 3.16 lists some special cases as the restrictions are imposed.
### Table 3.16

<table>
<thead>
<tr>
<th></th>
<th>Assumptions on Means</th>
<th>Assumptions on Volatilities</th>
<th>Assumptions on Correlations</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimal Mean-Variance</strong></td>
<td>Unconstrained</td>
<td>Unconstrained</td>
<td>Unconstrained</td>
<td>Most complex</td>
</tr>
<tr>
<td><strong>Minimum Variance</strong></td>
<td>Equal</td>
<td>Unconstrained</td>
<td>Unconstrained</td>
<td>No need to estimate means</td>
</tr>
<tr>
<td><strong>Risk Parity</strong></td>
<td>Equal</td>
<td>Unconstrained</td>
<td>Equal to zero</td>
<td>No need to estimate means or correlations</td>
</tr>
<tr>
<td><strong>Equally Weighted (1/N Portfolio)</strong></td>
<td>Equal</td>
<td>Equal</td>
<td>Equal</td>
<td>Most simple and active, Nothing to estimate</td>
</tr>
<tr>
<td><strong>Market Weight</strong></td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>Observable and passive, Nothing to estimate</td>
</tr>
</tbody>
</table>
The minimum variance portfolio is a special case of full mean-variance that does not estimate means and in fact assumes that the means are all equal. Risk parity is a special case of mean-variance that does not estimate means or correlations; it implicitly assumes that all assets have the same mean and all assets are uncorrelated. The equally weighted portfolio has nothing to estimate. It is also a special case of mean-variance and assumes all assets are identical.

I have included market weights in Table 3.16. Like equal weights, there are no parameters to estimate using market weights. The important difference between equal weights and market weights is that the equal weighted portfolio is an *active strategy*. It requires trading every period to rebalance back to equal weights. *(p.106)* In contrast, the market portfolio is *passive* and requires no trading. The action of rebalancing in equal weights imparts this strategy with a *rebalancing premium*. Rebalancing also turns out to be the foundation of an optimal long-run investing strategy. I cover these topics in the next chapter.

As you go down Table 3.16 from full-blown mean-variance to equal weights or the market portfolio, you estimate fewer parameters, and thus there are fewer things that can go wrong with the mean-variance optimization. The extreme cases are the equal weight or market weight positions, which require no analysis of data (except looking at market capitalizations in the case of the market portfolio).

Long-run means are very tricky to estimate. Sampling at weekly or daily frequencies does not allow you to more accurately estimate means—only extending the sample allows you to pin down the mean more precisely. For any asset like the S&P 500, the only way to gauge the long-run return is to look at the index level at the beginning and at the end of the sample and divide it by time. It doesn’t matter how it got to that final level; all that matters is the ending index value. Hence, we can only be more certain of the mean return if we lengthen time. This makes forecasting returns very difficult. The minimum variance portfolio outperforms mean-variance because we remove all the errors associated with means.

Volatilities are much more predictable than means. High-frequency sampling allows you to estimate variances more accurately even though it does nothing for improving
estimates of means. Higher frequency data also allows you to produce better estimates of correlations. But correlations can switch signs while variances can only be positive. Thus, variances are easier to estimate than correlations. Poor estimates of correlations also have severe effects on optimized mean-variance portfolios; small changes in correlations can produce big swings in portfolio weights. Risk parity turns off the noise associated with estimating correlations. (More advanced versions of risk parity do take into account some correlation estimates.) In the horserace, risk parity produced higher Sharpe ratios (0.59 and 0.65 using variances and volatilities, respectively) than the minimum variance portfolio (which had a Sharpe ratio of 0.52).

In summary, the special cases of mean-variance perform better than the full mean-variance procedure because fewer things can go wrong with estimates of the inputs.

5.3. Implications for Asset Owners

I went from the full mean-variance case to the various special cases in Table 3.16 by adding restrictions. To practice mean-variance investing, the investor should start at the bottom of Table 3.16 and begin from market weights. If you (p.107) can’t rebalance, hold the market. (The horserace results in Table 3.14 show that you will do pretty well and much better than mean-variance.)

If you can rebalance, move to the equal weight portfolio. You will do better than market weights in the long run. Equal weights may be hard to implement for very large investors because when trades are very large, investors move prices and incur substantial transaction costs. It turns out that any well-balanced, fixed-weight allocation works well. Jacobs, Müller, and Weber (2010) analyze more than 5,000 different portfolio construction methods and find that any simple fixed-weight allocation thrashes mean-variance portfolios.

Now if you can estimate variances or volatilities, you could think about risk parity. My horserace only estimated volatility by taking the realized volatility over a past rolling sample. Ideally we want estimates of future volatility. There are very good models for forecasting volatility based on generalized autoregressive conditional heteroskedasticity or stochastic volatility models, which I describe in chapter 8.
Suppose the hotshot econometrician you’ve just hired can also accurately estimate correlations and volatilities. Now you should consider relaxing the correlation restriction from risk parity. Finally, and hardest of all, is the case if you can accurately forecast means. If and only if you can do this, should you consider doing (fairly unconstrained) mean-variance optimization.

Common to all these portfolio strategies is the fact that they are diversified. This is the message you should take from this chapter. Diversification works. Computing optimal portfolios using full mean-variance techniques is treacherous, but simple diversification strategies do very well.

**Warning on Risk Parity**

The second panel of Table 3.14 reports the average weights in each asset class from the different strategies. Risk parity did very well, especially the risk parity strategy implemented with volatilities, because it overweighted bonds during the sample. Risk parity using variances held, on average, 51% in U.S. Treasuries and 36% in corporate bonds versus average market weights of 14% and 8%, respectively. There were even larger weights on bonds when risk parity is implemented weighting by volatilities rather than variances. Interest rates trended downward from the early 1980s all the way to the 2011, and bonds performed magnificently over this period (see chapter 9). This accounts for a large amount of the out-performance of risk parity over the sample.

Risk parity requires estimates of volatilities. Volatilities are statements of risk. Risk and prices, which embed future expected returns, are linked in equilibrium (see chapter 6). Howard Marks (2011), a hedge fund manager, says: “The value investor thinks of high risk and low prospective return as nothing but two sides of the same coin, both stemming primarily from high prices.” Risk parity overweights assets that have low volatilities. Past volatilities tend to be low precisely when today’s prices are high. Past low volatilities and high current prices, therefore, coincide with elevated risk today and in the future. At the time of writing, Treasury bonds have record low yields, and so bond prices are very high. Risk-free U.S. Treasuries can be the riskiest investment simply because of high prices. And at a low enough price, risky equities can be the safest investments. Risk parity,
Mean-Variance Investing

poorly implemented, will be pro-cyclical because it ignores valuations, and its pro-cyclicality will manifest over decades because of the slow mean reversion of interest rates.

6. Norway and Wal-Mart Redux
Diversification involves holding many assets. In section 2 we saw that when we started with the United States and progressively added countries to get to the G5 (United States, Japan, United Kingdom, Germany, France), there were tremendous benefits from adding assets. Conversely, if we go backward and remove assets from the G5, we decrease the diversification benefits.

Norway is excluding Wal-Mart on the basis of alleged violations of human rights and other ethical considerations. Removing any asset makes an investor worse off, except when the investor is not holding that asset in the first place. When we are forced to divest an asset, what is the reduction in diversification benefits?

6.1. Loss of Diversification Benefits
When I teach my case study on Norway and its disinvestment of Wal-Mart in my MBA Asset Management class, I ask the students to compute the lost diversification benefits from throwing out Wal-Mart. We can do this using mean-variance investing concepts. I will not do the same exercise as I give my students, but I will go through an experiment that removes various sectors from a world portfolio. This is also relevant to Norway because as of January 2010, Norway no longer holds any tobacco stocks. Other prominent funds including CalPERS and CalSTRS are also tobacco-free.

I take the FTSE All World portfolio as at the end of June 2012, when it had thirty-nine sectors and 2,871 stocks. What happens if we eliminate tobacco? Let’s use mean-variance concepts to quantify the loss of diversification benefits. In this exercise, I compute variances and correlations using a Bayesian shrinkage estimator operating on CAPM betas (see Ledoit and Wolf (2003)) and estimate expected (p.109) returns using a variant of Black-Litterman (1991). I set the risk-free rate to be 2%. I compute mean-variance frontiers constraining the sector weights to be positive.
We start with all the sectors. Then, I’ll remove tobacco. Next I’ll remove the aerospace and defense sector. Norway has selectively divested some companies in this sector because it automatically excludes all companies involved in the manufacture of nuclear weapons and cluster munitions. A final exclusion that I’ll examine is banks. Sharia law prohibits the active use of derivatives and debt as profit-making activities. Thus, it is interesting to see what diversification costs are routinely incurred by some Sharia compliant funds.

As we move from the full universe to the restricted universe, we obtain the following minimum standard deviations and maximum Sharpe ratios:

<table>
<thead>
<tr>
<th></th>
<th>Minimum Volatility</th>
<th>Maximum Sharpe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Sectors</td>
<td>0.1205</td>
<td>0.4853</td>
</tr>
<tr>
<td>No Tobacco</td>
<td>0.1210</td>
<td>0.4852</td>
</tr>
<tr>
<td>No Tobacco and Aerospace &amp; Defense</td>
<td>0.1210</td>
<td>0.4852</td>
</tr>
<tr>
<td>No Tobacco, Aerospace &amp; Defense, and Banks</td>
<td>0.1210</td>
<td>0.4843</td>
</tr>
</tbody>
</table>

The increase in the minimum volatility is tiny—from 12.05% to 12.10%. Similarly, the reduction in maximum Sharpe ratio is negligible, moving from 0.4853 for the full universe to 0.4852 when tobacco is removed and to 0.4843 when all three sectors are removed. Thus, the loss in diversification from removing one or a few sectors is extremely small. Figure 3.17 plots the (constrained) mean-variance frontiers for each set of sectors. They are indistinguishable on the graph. Norway is effectively losing nothing by selling Wal-Mart. It is also effectively losing nothing by excluding tobacco.
It is important to note that in this example that I am computing the loss of ex-ante diversification benefits, as measured by the minimum variance portfolio or the maximum Sharpe ratio portfolio—concepts that are relevant for an investor selecting from the FTSE sectors. I am not considering tracking error relative to the full FTSE universe or ex-post differences in returns (which is relevant for an investor forced to track the FTSE index).

(p.110) The extremely small costs of divestment are not due to the portfolios having zero holdings in the sectors that are being removed. In the full universe, the portfolio with the maximum Sharpe ratio contains 1.53% tobacco, 1.19% aerospace and defense, and 9.52% banks. Even removing an approximately 10% bank holding position has negligible cost in terms of diversification losses.

Diversification losses are so small because extra diversification benefits going from thirty-eight to thirty-nine sectors, or even thirty-six to thirty-nine sectors, are tiny (recall there are *decreasing marginal diversification benefits*). In section 2 when we added Germany and France to the G3 (United States, Japan, and United Kingdom), there was a much smaller shift in the frontier compared with moving from the United States–Japan to the G3 (see Figures 3.4 and 3.5). In our sector example the small marginal diversification benefits come about because there are few opportunities for that lost sector to pay off handsomely when the other thirty-eight sectors tank.

6.2. Socially Responsible Investing
From the mean-variance investing point of view, SRI must always lose money because it reduces diversification benefits. Could it make money as an active management (alpha) strategy? Studies like Kempf and Osthoff (2007) find that stocks that rank highly on KLD measures have high returns. MSCI, an index provider, considers various social and environmental criteria in ranking companies in its KLD indexes. While Norway has thrown Wal-Mart out, Wal-Mart gets high KLD ratings partly because it has taken many steps to reduce its carbon footprint. On the other hand, Geczy, Stambaugh, and Levin (2004) find that SRI mutual funds underperform their peers by thirty basis points per month. Harrison Hong, a Princeton academic and one of the leading scholars on SRI, shows in Hong and Kacperczyk (2009) that “sin” stocks like tobacco, firearms manufacturers, and gambling have higher risk-adjusted returns than comparable stocks.40

In his magnum opus written in 1936, Keynes says, “There is no clear evidence that the investment policy which is socially advantageous coincides with that which is most profitable.” My reading of the SRI literature is that Keynes’s remarks are equally applicable today.

I believe there is some scope for SRI in active management. There are some characteristics of firms that predict returns. Some of these effects are so pervasive that they are factors, as I discuss in chapter 7. Many of the firms that rank highly on SRI measures are likely to be more transparent, have good governance, senior managers who are less likely to steal, efficient inventory management, use few accounting gimmicks, and respond well to shareholder initiatives. These are all characteristics that we know are linked to firm performance. A simple example: limiting the rents managers can extract from shareholders allows shareholders to take home more. Gompers, Ishii, and Metrick (2003) create a governance index to rank companies from “dictatorships” to “republics.” Companies that have many provisions to entrench management, anti-takeover provisions, and limit proxy votes, for example, would be defined as dictatorships. They find that republics—which are also likely to rank high on SRI criteria—have higher returns than dictatorships.41
If you are able to pick firms based on particular properties and characteristics and these are related to SRI, then you might be able to outperform. This method of SRI does not throw out companies; it actively selects companies on SRI criteria but does not limit the manager’s investment opportunities by excluding companies.\textsuperscript{42} Like all active strategies, it is hard to beat factor-based strategies (see chapter 14).

(p.112) SRI also serves an important role when it reflects the preferences of an asset owner. In Norway’s case, practicing SRI gives the sovereign wealth fund legitimacy in the eyes of its owners—the Norwegian people.\textsuperscript{43} SRI is the asset owners’ choice, but the costs of imposing SRI constraints, or any other constraints, should be measured relative to the full investment set: how much are you giving up to be good?

The main message from mean-variance investing is to hold a diversified portfolio, which Norway does. Diversification benefits are a free lunch according to mean-variance investing. Doing SRI by exclusions is costly because it shrinks diversification benefits. But starting from a well-diversified portfolio (and some of the best-performing diversified portfolios are the most simple, like equal-weighted and market-weighted portfolios), the loss from excluding a few stocks is tiny. The cost of being socially responsible for Norway is negligible.

At the time of writing in 2013, Wal-Mart was still on Norway’s excluded list.

Notes:
\textsuperscript{(1)} This is based on “The Norwegian Government Pension Fund: The Divestiture of Wal-Mart Stores Inc.,” Columbia CaseWorks, ID#080301. The quote is from Ministry of Finance press release No. 44 in 2006.


\textsuperscript{(3)} Section 8 of the Government Pension Fund Regulation No. 123, December 2005.


(6) Although Antonio owns many ships, his wealth is held in only one asset class—venture capital—and so is undiversified across asset classes and is illiquid. Antonio should diversify across factors (see chapter 14). Unable to raise cash to lend to his friend, he is forced to go to Shylock for a loan, which he is forced to pay with a “pound of flesh” when he defaults. I cover asset allocation with illiquid assets in chapter 13.

(7) In this simple setting, there is only one period. In reality, correlations move over time and increase during bear markets. Ang and Bekaert (2002) show that international investments still offer significant diversification benefits under such circumstances. Christoffersen et al. (2013) report that correlations of international stock returns have increased over time but are still much lower in emerging markets than for developed ones. Investors also have access to frontier markets, which are the subset of the smallest, most illiquid, and least financially developed emerging markets.

(8) See Barber and Odean (2011).


(10) Individual investors, unfortunately, tend to do exactly the opposite—individuals hold larger amounts in employer stocks, which have had the strongest return performance over the last ten years, as Benartzi (2001) shows.

(11) See “Stay the Course? Portfolio Advice in the Face of Large Losses,” Columbia CaseWorks, ID #110309.

(12) Counting only private university endowments using NACUBO data at 2011 fiscal year end.

(13) See Karolyi and Stulz (2003) and Lewis (2011) for literature summaries.

(14) See also Asness, Israelov, and Liew (2011). Chua, Lai, and Lewis (2010), among others, argue that international diversification benefits have decreased over time hand in hand with greater integration, but this is offset to some extent by
the entrance of new international vehicles like emerging markets in the 1980s and frontier markets in the 2000s.

\(^{(15)}\) Technically it is movements in real exchange rates that matter. If purchasing power parity (PPP) held, then there would be no real exchange rate risk (see Adler and Dumas (1983)). There is a very active literature on PPP. PPP certainly does not hold in the short run, and academics have not reached consensus on whether PPP holds in the long run (twenty- to one-hundred-year horizons). Taylor and Taylor (2004) present a summary.

\(^{(16)}\) See Errunza, Hogan, and Hung (1999).

\(^{(17)}\) The first model along these lines was by Gehrig (1993). Recent models include Van Nieuwerburgh and Veldkamp (2009) where investors endogenously choose not to become informed about foreign equities in equilibrium.

\(^{(18)}\) The technical jargon for this is that higher moment risk measures are not necessarily subadditive. See Artzner et al. (1999).

\(^{(19)}\) Technically, you can Taylor expand a utility function so that the first term represents CRRA utility, which is approximately mean-variance utility.

\(^{(20)}\) The mathematical formulation of this problem corresponds to equation (3.3) with the constraint that the weights in risky assets sum to one or that the weight in the risk-free asset is equal to zero.

\(^{(21)}\) I have personally found this terminology a little confusing because “mean-variance efficient” portfolio sounds similar to “minimum variance” portfolio. Unfortunately this terminology is engrained, and I will also use it here.

\(^{(22)}\) The first paper in this literature is Blume, Crockett, and Friend (1974). The non-participation puzzle rose to economists’ attention with Mankiw and Zeldes (1991) as an explanation for the equity premium puzzle, which I discuss in chapter 8.

\(^{(23)}\) See Hong, Kubik, and Stein (2004) for social interaction, Bonaparte and Kumar (2013) for political activism, Guiso,


(25) See Ledoit and Wolf (2003) and Wang (2005), among others. Strictly speaking, the mean-variance solution involves an inverse of a covariance matrix, so we should shrink the inverse covariance rather than the covariance. This is done by Kourtis, Dotsis, and Markellos (2009). Tu and Zhou (2011) show shrinkage methods can be used to combine naïve and sophisticated diversification strategies in the presence of estimation risk.


(27) Qian, E., 2005, Risk Parity Portfolios: Efficient Portfolios through True Diversification, PanAgora.


(29) I cover hedge funds in chapter 17.

(30) Versions of risk parity where the weights are inversely proportional to volatility are advocated by Martellini (2008) and Choueifaty and Coignard (2008).


(32) At least since Haugen and Baker (1991).

(33) For N assets, you have N means and \(N \times (N + 1)/2\) elements in the covariance matrix.

(34) This is shown in a seminal paper by Merton (1980).

(35) I cover this further in chapter 8.
(36) Green and Hollifield (1992) provide bounds on the average correlation between asset returns that are required for portfolios to be well balanced.

(37) A contrary opinion is Asness, Frazzini, and Pedersen (2012). They argue that investors are averse to leverage and this causes safe assets to have higher risk-adjusted returns than riskier assets. Risk parity allows some investors to exploit this risk premium.


(39) This does not include the actual transaction costs of divestment. My case “The Norwegian Government Pension Fund: The Divestiture of Wal-Mart Stores Inc.,” Columbia CaseWorks, ID#080301, also estimates these transaction costs.

(40) Hong, Kubik, and Scheinkman (2012) argue against the hypothesis of “doing well by doing good” and argue exactly the opposite. They show that corporate social responsibility is costly for firms and firms only do good when they are not financially constrained. In this sense, corporate social responsibility is costly for firms.

(41) There is debate about whether this effect has persisted after the original Gompers, Ishii, and Metrick (2003) study and whether the effect is about risk or mispricing (see chapter 7). Cremers and Ferrell (2012) find stocks with weak shareholder rights have negative excess returns from 1978 to 2007 while Bebchuk, Cohen, and Wang (2013) argue the original Gompers, Ishii, and Metrick results disappear in the 2000s.

(42) A more aggressive form of doing this is shareholder activism. Shareholder activism by hedge funds adds significant value (see Brav et al. (2008)) even though the evidence for mutual funds and pension funds adding value for shareholders is decidedly mixed (see Gillan and Starks (2007)). Dimson, Karakas, and Li (2012) find that corporate social responsibility activist engagements generate excess returns.
(43) See Ang (2012a).