Journal of King Saud University – Computer and Information Sciences (2019) 31, 161–174

King Saud University

Journal of King Saud University – Computer and Information Sciences

www.ksu.edu.sa
www.sciencedirect.com

Local appearance-based face recognition using adaptive directional wavelet transform

Mohd. Abdul Muqeet [a,*], **Raghunath S. Holambe** [b]

[a] *Electrical Engineering Department, Muffakham Jah College of Engineering and Technology, Hyderabad (TS), India*
[b] *Department of Instrumentation Engineering, SGGS Institute of Engineering and Technology, Nanded (MS), India*

Received 1 September 2016; revised 30 November 2016; accepted 31 December 2016
Available online 9 January 2017

KEYWORDS

Face recognition;
Adaptive directional wavelet transform;
Linear discriminant analysis;
Multiresolution analysis

Abstract The latest research has shown that adaptive directional wavelet transform can constitute edges and textures in images efficiently due to the adaptive directional selectivity. This paper is primarily focused on the application of adaptive directional wavelet transform in conjunction with linear discriminant analysis (LDA) for capturing the discriminant directional multiresolution facial features. The intention of this paper is to explore the efficacy of adaptive directional wavelet transform in facial feature extraction and to offer a stepping stone for further research in this direction. The proposed approach is compared with existing subspace and local descriptor feature extraction methods. A performance comparison is also demonstrated with existing non-adaptive multiresolution analysis methods such as discrete wavelet transform (DWT), Gabor wavelet transform (GWT), curvelets, ridgelets, contourlets, and local Gabor binary pattern. Evaluation of the proposed approach on famous databases such as ORL, Essex Grimace, Yale, and Sterling face convinces the effectiveness of the adaptive directional wavelet transform based subspace features.

1. Introduction

Face as a biometric pattern has been broadly utilized for diverse applications such as personality identification, security checking, suspect pursuit and video surveillance. The primary assignment of face recognition is to identify the face image against face images stored in a face database. Face recognition methods are predominantly influenced by challenges such as variation in illumination, expression, head pose variation and occlusion (Jain et al., 2006). Holistic subspace methods such as principal component analysis (PCA) (Turk and Pentland, 1991), kernel PCA (Schoelkopf et al., 1997), independent component analysis (ICA) (Bartlett et al., 1998), linear discriminant analysis (LDA) (Belhumeur et al., 1997), and kernel LDA (Juwei et al., 2003) are successfully used for facial feature extraction. PCA and LDA are statistical methods and experience generalization problem in real time environments. Local binary pattern (LBP) (Ahonen et al., 2006) being a non-statistical face recognition method outperforms PCA and LDA-based face recognition methods in terms of recognition performance and computational simplicity. The shortcoming of LBP-based face recognition methods is their

* Corresponding author.
E-mail address: ab.muqeet2016@gmail.com (M.A. Muqeet).
Peer review under responsibility of King Saud University.

ELSEVIER Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.jksuci.2016.12.008
1319-1578

sensitivity to noise and a massive length of the feature vectors. Weber local descriptor (WLD) (Chen et al., 2010) is a simple but efficient local descriptor compared to LBP. Li et al. (2013) presented a face recognition method using multiscale WLD to describe the facial images and modified the original approach by a non-linear quantization approach to enhance its discriminative power. In (Lei et al., 2015) an efficient feature extraction algorithm called discriminant sparse local spline embedding is proposed. Experimental results are demonstrated on ORL and Yale face database to explain its effectiveness for face recognition. Cao et al. (2016) applied a multi-pose sparse representation algorithm to address face recognition under pose and illumination variation.

The two-dimensional discrete wavelet transform (DWT) (Sweldens, 1998; Daubechies and Sweldens, 1998) is an isotropic multiresolution analysis (MRA) method which provides edge information along the horizontal and vertical directions. Current interests in the improvement of face recognition methods have grown in the application of multiresolution analysis (MRA) methods such as wavelets (Feng et al., 2000; Chien and Wu, 2002; Muqeet and Holambe, 2016; Abdul Muqeet, 2016), Gabor wavelets (Liu and Wechsler, 2002; Liu, 2004; Struc and Pavesic, 2009; Vinay et al., 2015), curvelets (Candes et al., 2007), ridgelets (Do and Vetterli, 2003), contourlets (Do and Vetterli, 2005), and steerable pyramid transform (Simoncelli, 1996). Huang et al. (2015a) proposed a face recognition technique of taking the joint of pixel-level and feature-level fusion of top level wavelet subbands. In Huang et al. 2015b a non-uniform patch based strategy is used for high-frequency subbands to successfully reflect the structure of the face image. Nonetheless, due to the isotropic scaling of DWT, extracted wavelet features do not provide additional edge relevant information from face images. GWT is computationally complex due to the convolution of each face image with different Gabor kernels described by the values of scale and orientation (Liu, 2004). A fast discrete curvelet based face recognition method with PCA and LDA as dimensionality reduction method has been proposed by Mandal et al. (2009). Curvelet transform is integrated with support vector machine with particle swarm optimization method for facial expression recognition system (Tang and Chen, 2013). Boukabou and Bouridane (2008) applied contourlet transform on face images of Yale and FERET databases and used PCA as dimensionality reduction method. Jadhav and Holambe, 2008 applied the radon transform to obtain radon space face features and then applied wavelet transform on these radon features to develop a facial expression and illumination invariant face recognition system. Kautkar et al. (2012) applied finite ridgelet transform (FRIT) to 2D/2.5D face images and used the fisherfaces as the dimensionality reduction method. Steerable pyramid (S-P) transform face recognition framework is proposed in (El Aroussi et al., 2011). S-P based features present better results compared with PCA, LDA and some multiresolution methods including DWT, GWT, curvelets, and contourlets. Another multiresolution approach is to combine Gabor filters with traditional LBP to obtain an efficient local descriptor called local Gabor binary pattern (LGBP) (Zhang et al., 2005). The resulting LGBP leads to good results in facial recognition but at the cost of the large size of the feature vector.

In all the aforementioned MRA methods, the edge related directional data from face images is captured in a non-adaptive way. If this directional data and orientation of the edges from a face image can be acquired adaptively it will considerably improve the overall performance of the face recognition method. Multiresolution analysis methods that adaptively select the filtering direction explicitly via lifting scheme were proposed by (Chang and Girod (2007), Ding et al. (2007), Maleki et al. (2012)). These methods extend the present lifting wavelet transform with better directionality though still retaining its structure and desirable features. Chang and Girod (2007) proposed a direction adaptive discrete wavelet transform (DA-DWT) which entails the application of the directional lifting based on the content of the images. Essentially, the method in Chang and Girod (2007) is proved to be efficient for image compression and suggests improvement in comparison to traditional lifting based approaches. The other famous work based on the adaptation of the filtering direction in lifting scheme is proposed by Ding et al. (2007). The principal difference between the works of Chang and Girod (2007) and Ding et al. (2007) is that in Chang and Girod (2007) the lifting wavelet transform is realized simply with one pair of lifting steps and interpolating Neville filters (Kovacevic and Sweldens, 2000) are used as the prediction and update filters. DA-DWT (Chang and Girod, 2007) also does not involve subsample interpolation and next integer samples are considered which are farther away from the prediction samples. Due to the improvements offered by adaptive directional lifting schemes (Chang and Girod, 2007 and Ding et al., 2007) in image analysis, and their capability in approximating the directional data, this paper utilizes the concept of direction adaptive DWT (DA-DWT) Chang and Girod, 2007 to investigate its efficacy in a face recognition framework.

In this paper, we propose a new face recognition approach based on DA-DWT and LDA to extract expression, illumination, and pose invariant discriminant multiresolution features. To avoid the confusion with the original DA-DWT work (Chang and Girod, 2007) we will name it as adaptive directional wavelet transform (ADWT). The contributions of the proposed approach are: (1) applying the adaptive directional wavelet transform (ADWT) with the sub-block partitioning and optimum direction selection as concurrent methods to acquire improved directional multiresolution facial features replicated in low frequency subband (2) applying LDA subspace to the low frequency subband to obtain reduced dimension discriminant facial features. Renowned face databases such as ORL (Available: http://www.uk.research.), Essex Grimace (Available:http://cswww.essex.ac.uk/mv/allfac), Yale (Available: http://cvc.yale.edu/pro), and Sterling face (University of Stirling online database) are used for performance experimentation purpose. Performance comparison is investigated by comparing the proposed results with numerous existing face recognition methods alongside some non-adaptive multiresolution analysis (MRA) methods. The remainder of the paper is structured as follows. In Section 2 the proposed face recognition system is described. Experimental results are presented in Section 3. Section 4 concludes the paper.

2. Proposed face recognition approach

In this section, we describe the basic proposal of the proposed approach. The proposed face recognition method is essentially composed of three main phases. In the first phase, we apply 2-D ADWT along with sub-block partitioning algorithm to extract directional multiresolution features located in low-frequency approximation LL subband. In the second phase LDA is applied on the LL subband to obtain most discriminant multiresolution features. In the final step, classification using the nearest neighbor classifier is performed and performance parameters are derived.

2.1. 2-D adaptive directional wavelet transform (2-D ADWT)

Rectilinear 2-D discrete wavelet transform is effectively implemented using one or two pairs of lifting scheme (Sweldens, 1998) (Daubechies and Sweldens, 1998). The rectilinear lifting scheme performs the prediction and update operations only in neighbor horizontal or vertical direction. Owing to the efficient adaptive filter direction selection, the method in Chang and Girod (2007) can efficiently approximate directional image features. Following the notation given in Chang and Girod (2007) let X denote an image defined on an orthogonal sampling grid $\prod = \{(m,n) \in^2 | 0 \leqslant m \leqslant M-1, 0 \leqslant n \leqslant N-1\}$.

This sampling grid is composed of 4 subgrids $\prod = \cup \prod_{pq}$, with $\prod_{pq} = \{(m,n) | m\text{mode}2 = p, n\text{mode}2 = q\}, p,q \in \{0,1\}$.

Similar to the rectilinear DWT the input image is first decomposed into even rows $X_0 = \{X[l_0], l_0 \in \prod_0 = \prod_{00} \cup \prod_{01}\}$ and odd rows $X_1 = \{X[l_1], l_1 \in \prod_1 = \prod_{10} \cup \prod_{11}\}$. The lifting scheme predicts the even rows from the odd rows resulting in high-pass subband $H = \{H[l_1]\}$ defined on $\prod_1$. The resulting high-pass subband is used to update the even rows in order to produce the approximation low-frequency subband $L = \{L[l_0]\}$ defined on $\prod_0$. The 1-D lifting transform can be expressed as (Chang and Girod, 2007):

$$H[l_1] = g_H \cdot (X[l_1] - P_{X,l_1}(X_0)), \forall l_1 \in \Pi_1 \tag{1}$$

$$L[l_0] = g_L \cdot \left(X[l_0] - g_H^{-1} \cdot U_{X,l_0}(H)\right), \forall l_0 \in \Pi_0 \tag{2}$$

where P_{X,l_1} and U_{X,l_0} are the prediction and update functions respectively taking as input sets of samples in X_0 and H respectively. g_H and g_L are the scaling factors. In the prediction step described in (1), for each sample in the high-pass subband $H = \{H[l_1]\}$, it is desirable to select prediction function P_{X,l_1} that predicts odd rows $X[l_1]$ from neighboring even rows X_0 such that the magnitude of the high-pass subband $H[l_1]$ is minimized. In our proposed approach we selected nine possible candidate directions N_c from which prediction and update steps can be adaptively performed (Chang and Girod, 2007):

$$P_{X,l_1}(X_0) = \sum_{k=-K^p}^{K^p-1} c_k^p \cdot (X[l_1 - (2k+1)v_i] + X[l_1 + (2k+1)v_i]) \tag{3}$$

$$U_{X,l_0}(H) = \sum_{k=-K^u}^{K^u-1} c_k^u \cdot \left(\sum_{l_1 | l_1 - (2K+1)\bar{v}_{l_1} = l_0} H[l_1] + \sum_{l_1 | l_1 + (2K+1)\bar{v}_{l_1} = l_0} H[l_1] \right) \tag{4}$$

where $2K^p$, and c_k^p are the length and coefficients of the prediction filter respectively. Similarly $2K^u$, and c_k^u are the length and coefficients of the update filter respectively.

In the above equations $i = 0, \ldots N_c - 1$ is the direction index and each candidate should correspond to perform lifting wavelet transform along a particular direction defined by v_i. The $v_i s$ are defined such that all $l_1 + (2k+1)v_i$ map to an even l_0 index. From (4) whenever an image sample $X[l_i]$ is predicted by $c_k^p X[l_o]$, $L[l_0]$ is updated by $c_k^u H[l_1]$. In the proposed approach we considered the candidate directions (Chang and Girod, 2007) with $N_c = 9$ and $v_0 = (0,1)$, $v_1 = (1,3)$, $v_2 = (1,1)$, $v_3 = (2,1)$, $v_4 = (3,1)$, $v_5 = (-3,1)$, $v_6 = (-2,1)$, $v_7 = (-1,1)$, $v_8 = (-1,3)$. These candidate directions correspond to the direction set given as

$$\Theta = \{\theta | \theta = 0, \pm 18.43, \pm 45, \pm 63.43, \pm 71.57\} \tag{5}$$

A subsequent second-dimensional lifting step is performed on columns of $L[l_0]$ and $H[l_1]$ with a different direction set $u_i = (u_{i,m}, u_{i,n})$ which is defined such that $l_{01} \pm (2k+1)u_i \in \Pi_{00}$ and $l_{11} \pm (2k+1)u_i \in \Pi_{10}$, with $k = 0, \ldots K^p - 1$. The prediction and update functions are similarly defined as given in (3) and (4).

The different candidate directions N_c^* defined for second-dimensional lifting are $u_0 = (1,0)$, $u_1 = (3,2)$, $u_2 = (1,2)$, $u_3 = (1,4)$, $u_4 = (1,6)$, $u_5 = (1,-6)$, $u_6 = (1,-4)$, $u_7 = (1,-2)$, $u_8 = (3,-2)$. The candidate directions in the second dimensional lifting corresponds to the direction set given as,

$$\Theta^* = \{\theta^* | \theta^* = 90, \pm 56.31, \pm 26.57, \pm 14.04, \pm 9.46\} \tag{6}$$

We also define symmetrical extension at the boundaries such as $X[-1,0] = X[1,0]$ to account for the samples locations not defined in Π (Chang and Girod, 2007). For face recognition problem we proposed to use prediction filter coefficients with eight vanishing moments i.e. $K^p = 4$ with $c_0^p = c_{-1}^p = 1225/2048$, $c_1^p = c_{-2}^p = -245/2048$, $c_2^p = c_{-3}^p = 49/2048$, and $c_3^p = c_{-4}^p = -5/2048$, and update filter coefficients with eight vanishing moments i.e. $K^u = 4$ with $c_0^u = c_{-1}^u = 1225/4096$, $c_1^u = c_{-2}^u = -245/4096$, $c_2^u = c_{-3}^u = 49/4096$, and $c_3^p = c_{-4}^p = -5/4096$. In our proposed approach we used (8,8) interpolating wavelet (Kovacevic and Sweldens, 2000) as opposed to the original work in (Chang and Girod, 2007) where the prediction and update filters with (6,6) interpolating filters are used. We selected these (8, 8) interpolating wavelet (Kovacevic and Sweldens, 2000), as they result in the best verification performance in our experiments. The interpolating filters also hold linear phase characteristics (Kovacevic and Sweldens, 2000) which extend their texture discrimination potential and hence can be effectively utilized in facial feature extraction. Rather than continually making a prediction and update process in the horizontal or vertical direction, the 2-D ADWT chooses a direction, and the prediction and update processes are performed in these directions to reduce the energy summation of the prediction error (*ESPE*). The value of *ESPE* accounts for the amount of variance present in the high-frequency subband. While a sample is predicted from neighboring samples, each candidate direction is checked and the direction with the smallest *ESPE* is finally selected.

2.2. Sub-block partitioning and direction estimation from face image

The main function of the proposed approach is to approximate significant directional information from face images. To successfully achieve this locally, it is essential to adaptively partition the face image into variable size blocks of apparent edge related directional details. In our experiments for face feature extraction, a sub-block partitioning scheme (Chang and Girod, 2007) is used to partition the face image in non-overlapping blocks and filtering directions may be selected from each block based upon the minimum value of the *ESPE*. The features which are oriented in these directions are estimated as dominant multiresolution facial features. The prediction and update lifting steps as given in (3) and (4) are carried out in this selected direction within this local sub-block. The task of selection of such sub-block and the corresponding optimal direction is executed effectively by sub-block partitioning algorithm as mentioned in Algorithm 1. Sub-block partitioning is controlled by the value of the Lagrangian multiplier. This value is wisely selected for different face database as mentioned in Section 3.3. As mentioned in (Chang and Girod, 2007) for image compression the filtering directions are selected to reduce the distortion in the reconstructed image, whereas for face recognition we selected the filtering directions to reduce the variance in the high-frequency subbands. This adaptation significantly concentrates entire image energy in low-frequency subband and in turn reduces the energy of high-frequency subbands. Fig. 1(a) shows the resulting partitions and selected directions for 2-D DWT and Fig. 1(b) shows the resulting partitions and direction for 2-D ADWT on the sample face image of Sterling face database. Note that even though the direction is selected block-wise, filtering in the prediction and update step extends across block boundaries (Chang and Girod, 2007) as shown in Fig. 1(b).

Algorithm 1. Sub-block partitioning algorithm. For all the images available in the database do step 1 to step 7.

Input	Face image, initialize sub-block size S_{ini}, value of the Lagrangian multiplier λ.
Output	High-pass and low-pass wavelet subbands.
Step 1	Block partition each face image X defined on the sampling grid Π is evenly partitioned into non-overlapping sub-blocks, each sub-block denoted by N_b of size S_{ini}.
Step 2	If $S_{ini} \geqslant 4$ then partition N_b into $2\times1, 1\times2, 2\times2, 4\times1, 1\times4, 4\times2, 2\times4, 4\times4$ sub-blocks.
Step 3	For each sub-block N_b calculate the filtered directional response R_b along each direction θ.
Step 4	Calculate the energy summation of the prediction error from the filtered directional responses $R_{b,\theta}$, denoted as $ESPE(N_b^\theta) = \{\lvert\lvert\sum_{i=1}^{M}\sum_{j=1}^{N} R_{b,\theta}(i,j)\rvert\rvert^2 + \lambda D^\theta\}.D^\theta$ is the number of bits spent on signaling the selection of directions. M and N are the size of the respective sub-blocks.
Step 5	For each sub-block N_b calculate the optimal direction along which the minimum value of energy summation of prediction error is obtained $\theta_b = \arg\min_\theta\{ESPE(N_b^\theta)\}$.
Step 6	Apply the prediction and update lifting steps in the selected sub-block in the selected direction obtained in Step 5 and Step 6.
Step 7	Obtain the high-pass and low-pass wavelet subbands as given in (1) and (2).

2.3. Feature extraction

The feature extraction process of the proposed approach utilizing 2-D ADWT also involves some parameter settings like the selection of the size of each sub-block and value of the Lagrangian multiplier λ which controls the complexity of sub-block partitioning. The optimal size of each sub-block S_{ini} and value of the Lagrangian multiplier λ influence the performance of our proposed approach which is discussed in detail in Section 3.3. The direction adaptation along with lifting-based wavelet implementation considerably concentrates entire image energy in low-frequency subband and reduces the energy of high-frequency subbands. In our proposed approach we selected two decomposition levels for all the face databases. Fig. 2 illustrates the process-flow of the proposed face recognition approach. We selected the top-level lowest frequency subband LL as the extracted multiresolution feature, as it represents the main structure of a face image and consists of maximum energy as compared to other subbands.

2.4. Dimensionality reduction and classification

Thus selecting the low-frequency LL subband reduces the dimension of the original image of a size which is represented as a vector from 1×16384 to 1×1024. To again reduce the dimensionality of the feature vector and to achieve discriminant features LDA (Belhumeur et al., 1997) is applied. LDA is a popular discriminant method which measures the between-class scatter normalized by the within-class scatter (Belhumeur et al., 1997). Given the set of feature vectors $F = [f_1, f_2, \ldots, f_n]$, obtained from the previous step each belonging to one of the N classes $\{c_1, c_2, \ldots, c_N\}$, the between-class and within-class scatter matrices $\sum_b$ and $\sum_w$ are defined as follows (Belhumeur et al., 1997):

$$\sum_b = \sum_{i=1}^{N} n_i \cdot (\mu_i - \mu)\cdot(\mu_i - \mu)^T \tag{7}$$

$$\sum_w = \sum_{i=1}^{N}\sum_{f_j\in c_i}(f_j - \mu_i)(f_j - \mu_i)^T, \tag{8}$$

where N is the total number of classes, μ_i is the mean of the classes c_i, n_i is the number of samples in ith class, and μ stands for global mean. LDA drives a transformation matrix $\mathbf{W}$ which maximizes the ratio (Belhumeur et al., 1997):

$$J(W) = \arg\max{}_W \frac{\lvert W^T \sum_b W\rvert}{\lvert W^T \sum_w W\rvert} \tag{9}$$

This ratio is maximized when $\mathbf{W}$ consists of eigenvectors w_i of the matrix $\sum_w^{-1}\sum_b$ (Belhumeur et al., 1997):

$$\sum_w^{-1}\sum_b w_i = \lambda_i w_i, \quad i = 1, 2, \ldots, N-1 \tag{10}$$

To avoid singularity issues, while computing $\sum_w^{-1}$, LDA subspace is implemented in PCA subspace (Belhumeur et al., 1997). The implementation of LDA over 2-D ADWT multiresolution features provides the most discriminant class-specific features. Then the resultant features are finally set for classification. A simple nearest neighbor (NN) classifier is used to

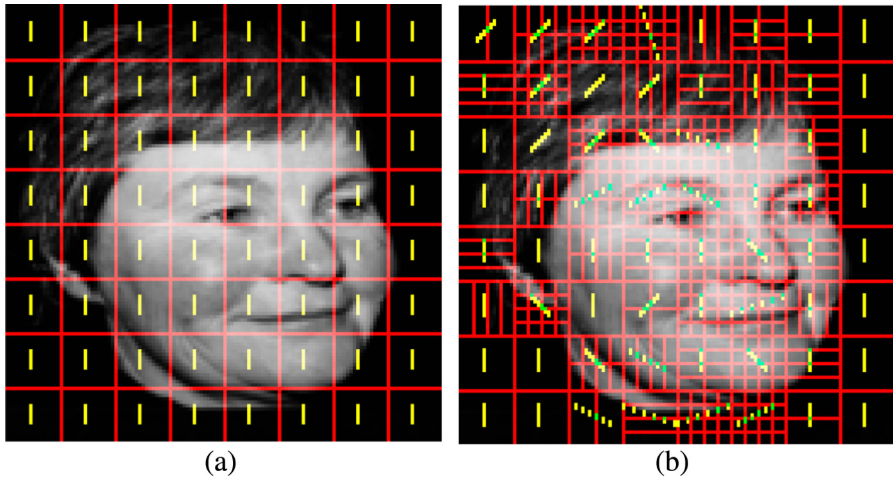


Figure 1 (a) Sub-block partitioning structure and the selected directions for 2-D DWT (b) Sub-block partitioning structure and the selected directions for 2-D ADWT.

perform the face verification process. In our experiments, we used cosine distance measure for PCA and KPCA-based methods (Struc and Pavesic, 2009; Vinay et al., 2015) and the Euclidean distance measure for LDA-based methods to compute the distance between the resulting feature vectors and the probe image feature vector.

3. Experimental results and discussion

In this section, we evaluate the performance of the proposed approach using four databases, Olivetti Research Laboratory (ORL), Essex Grimace, Yale face, and Sterling face databases. The experiments were carried out in Matlab 2014a, on a 64-bit I3, 2.13 GHz processor, with 2 GB RAM. The effectiveness of this method is compared in terms of performance parameters like false acceptance rate (FAR), false rejection rate (FRR), and verification accuracy against some existing popular subspace face recognition methods such as PCA (Turk and Pentland, 1991), wavelet-based PCA (WPCA) Feng et al., 2000, KPCA (Liu, 2004), LDA (Belhumeur et al., 1997), wavelet-based LDA (WLDA) (Chien and Wu, 2002), and KLDA (Juwei et al., 2003). We also compared the performance of the proposed approach against some local descriptor methods such as LBP (Ahonen et al., 2006), and WLD (Li et al., 2013). Additionally a performance comparison is also provided against some famous non-adaptive directional multiresolution analysis methods used for face recognition such as Gabor fisher classifier (GFC) (Liu and Wechsler, 2002), curvelet transform (Mandal et al., 2009), contourlet transform (Boukabou and Bouridane, 2008), ridgelet transform (Kautkar et al., 2012), and LGBP (Zhang et al., 2005). In the preliminary experiments, we selected the values of parameters essential for performing the proposed approach for getting better results as indicated in Section 3.2. These parameters include the optimal initial block size for each face image and value of the Lagrangian multiplier. The following sets of experiments were carried out

1. Testing the performances of methods in facial expression variations.
2. Testing the performances of methods in illumination variations.
3. Testing the performances of methods for pose variations.

3.1. Face databases

To investigate the usefulness of the proposed approach for expression, illumination, and pose variation we considered well-known databases such as ORL, Essex Grimace, Yale, and Sterling face. The ORL database (Available: http://www.uk.research) consists of 10 dissimilar face images of 40 distinct

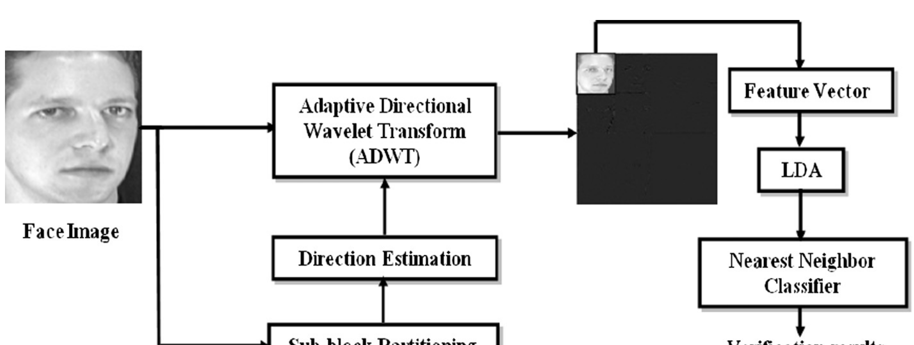


Figure 2 Process-flow diagram of the proposed approach (ORL database face image).

persons. There are variations in the capture time, lighting, head position, facial expressions such as eyes open or closed, smiling or not smiling and facial particulars such as glasses or no glasses. All images consist of 256 gray levels with a resolution of 112×92 pixels captured in a dark consistent background. For experimentation, the face images are resized to a resolution of 128×128 pixels (Fig. 3). The Essex Grimace database (Available:http://cswww.essex.ac.uk/mv/allfac) consists of 20 face images each for 18 persons inclusive of male and female, captured against a plain background with a fixed camera. Each image is of resolution size 180×200 pixels. Face images comprise of grimaces, slight variation in illumination and head scale, major variation in expression. For experimentation, we converted the RGB images to grayscale images and resized to 256×256 pixel resolution (Fig. 4). The Yale face database (Available: http://cvc.yale.edu/pro) includes face images of 15 persons. There are 11 face images for each person with illumination and facial expression variations. All the images are resized to 128×128 pixel resolution (Fig. 5). Sterling face database (University of Stirling online database) consists a total of 312 monochrome images of 18 females and 17 males, in 3 different poses and expressions. Each face image is of resolution 269×369. For experimentation purpose, we consider 33 subjects with 9 images for each subject and each image is resized to a 256×256 pixel resolution (Fig. 6).

3.2. Performance measures

The performance of the proposed approach is assessed by evaluating the error rates. The error rate obtained by rejecting authentic users with respect to different values of threshold (Th) is called false rejection rate (FRR), and accepting imposter users with respect to different values of threshold (Th) is called false acceptance rate (FAR). False rejection (FR) only occurs when an authentic user is rejected when the classification test fails to recognize correctly i.e., say Euclidean distance (ED) is greater than their corresponding threshold, i.e., $FR = \{ED_i^I > Th_i\}$, where superscript I denotes the intra-class comparisons. The final FRR is computed by counting the number of ED_i^I greater than Th_i denoted as $Cnt(ED_i^I, Th_i)$. It is expressed as,

$$FRR = \prod_{i=1}^{k} \frac{Cnt\left(ED_i^I > Th_i\right)}{N_k} \tag{11}$$

where N_k is the total number of intra-class comparisons. The false acceptance (FA) occurs when during the classification the ED is less than or equal to the corresponding threshold, i.e., $FR = \{ED_i^E \leqslant Th_i\}$, where superscript E denotes the inter-class comparisons.

The final FAR is computed by counting the number of ED_i^E to be less than or equal to Th_i denoted as $Cnt(ED_i^E, Th_i)$. It is expressed as,

Figure 3 Sample face images of ORL database.

Figure 4 Sample face images of Essex Grimace database.

Figure 5 Sample face images of Yale face database.

Figure 6 Sample face images of Sterling face database.

$$FAR = \prod_{i=1}^{k} \frac{Cnt(ED_i^I \leqslant Th_i)}{M_k} \tag{12}$$

where M_k is the total number of inter-class comparisons. The genuine acceptance rate (GAR) is a measure to calculate the percentage of genuine users authenticated by the system and can be defined as,

$$GAR(Th_i) = 1 - FRR(Th_i) \tag{13}$$

A low value of FAR and FRR is often enviable but as the FAR increases FRR decreases, this is due to the uneven distribution of genuine and impostor user scores. Thus equal error rate (EER) is calculated for different values of threshold to find an optimum balance between FAR and FRR. The EER is considered as the point where $FAR(Th_i) = FRR(Th_i)$. For a given threshold (Th_i), the average error is defined as $0.5.[FAR(Th_i) + FRR(Th_i)]$. If Av is the minimum average error for different thresholds, then verification accuracy is defined as

$$Verif \cdot Acc = (100 - Av)\% \tag{14}$$

In our experimentation, we followed the work of (Jin et al., 2004) for the FAR and FRR computations. For the FAR test, the first image of each face is matched against the first image of all other faces and the same matching procedure is repeated for subsequent face images. For FRR test, each image of each face is matched against all other images of the same face. It is desirable to reduce the intra-class and inter-class variations in the face recognition method. In this paper, receiver operating curve (ROC) (Jonsson et al., 2002) is generated to illustrate the discrimination capability of the proposed approach. ROC plots the genuine acceptance rate (GAR) as opposed to the false acceptance rate (FAR).

3.3. Selection of optimal parameters

To obtain the optimal value of the Lagrangian multiplier λ and the optimal value of initial sub-block size S_{ini}, we considered two distinctive sub-block sizes such as 8×8 and 16×16, and estimated the recognition accuracies for three different values for the Lagrangian multiplier as proved in Table 1. Thus for $\lambda = 5$ and sub-block size as 8×8 we achieved higher verification accuracies for ORL and Essex Grimace databases. For Yale database and Sterling face database, with $\lambda = 3$ and sub-block size as 8×8 we achieved higher verification accuracies. We preferred these values in our proposed approach to acquire dominant feature vectors. For our experiments, the scaling factors as mentioned in (1) and (2) are selected as $g_H = 0.7912$ and $g_L = 1.3061$ (Chang and Girod, 2007).

We additionally provided a comparison with diverse direction sets to prove the effectiveness of selected direction set with (8, 8) vanishing moments prediction and update filters. We

Table 1 Selection of sub-block size and Lagrangian multiplier for databases.

Database	$\lambda = 3$		$\lambda = 5$		$\lambda = 7$	
	8×8	16×16	8×8	16×16	8×8	16×16
ORL	98.39	98.84	99.58	99.08	99.17	98.84
Essex Grimace	98.43	98.13	99.32	98.58	98.72	98.38
Yale	99.41	99.19	98.64	99.31	99.09	98.09
Sterling faces	99.06	97.59	98.86	97.76	97.74	98.07

Table 2 Comparison of different directions set with respect to *EER* values.

Database	Direction set with nine directions with interpolated samples Ding et al. (2007)	Direction set with five directions Maleki et al. (2012)	Direction set with nine directions and (8,8) vanishing moments proposed approach
ORL	0.67	0.52	0.42
Essex Grimace	0.94	1.23	0.68
Yale	1.04	0.68	0.59
Sterling face	2.17	1.89	0.94

compared it with the nine directions given in (Ding et al., 2007), and five directions given in (Maleki et al., 2012) with respect to performance parameters. Compared to these directions, the nine directions with interpolating Neville filters with (8, 8) vanishing moments achieve higher performance results as specified in Table 2.

3.4. Experimental results

3.4.1. Testing the performances of the methods in facial expression variations

The first set of experiment compares the performance of the proposed approach with some popular existing face recognition methods for face images with facial expressions. In this experiment, we consider ORL and Essex Grimace face database images. Face verification experiment conducted on ORL face database images resulted in 2200 genuine attempts and 156,400 impostor attempts. While for Essex Grimace database, face verification results in 3780 genuine attempts and 122,760 impostor attempts.

(a) Subspace and local descriptor methods

To emphasize the discriminant strength of the proposed approach, a performance comparison with PCA, KPCA, KLDA, wavelet-based PCA (WPCA), LDA, wavelet-based LDA (WLDA), LBP, and WLD is shown in Table 3 for ORL and Essex Grimace databases. Considering the number of face images in these databases, the maximum length of the feature vector after dimensionality reduction for PCA, KPCA, and WPCA is 399 for ORL, and 359 for Essex Grimace database. The maximum feature vector length after dimensionality reduction for LDA, KLDA, and WLDA for ORL, and Essex Grimace database is 39, and 17 respectively. For LBP-based face recognition method, we apply rotation invariant uniform pattern *riu2* with 16 pixels in a circle of radius 2. The rectilinear 2-D DWT only considers the horizontal and vertical directions and consequently fails to approximate edges with prominent directional features which are neither horizontal nor vertical. This distributes the energy of such directional features in the high-frequency subband. Because of 2-D ADWT method and its associated adaptive directional selection process, these prominent directional edge features are approximated effectively. The optimal sub-block size and the Lagrangian multiplier value are elegantly decided in selecting a sub-block with essential directional features. Additionally, considering direction set given in (5) for the first-dimensional lifting and considering direction set given in (6) for the second-dimensional lifting cover a wide area of image pixels for capturing the directionality of face image characteristics. Accordingly, the adaptive directional selectivity and related prediction and update operations performed alongside these directions reduce the energy in high-frequency subband and concentrate maximum energy in low-frequency subband. Application of LDA on this low-frequency subband additionally provides most discriminant multiresolution features. For ORL database LBP provides close verification accuracy of 99.35% but with higher inter-class and intra-class variations as compared to our proposed approach. For Essex Grimace database WLD provides close verification accuracy of 99.12%. Thus for the expression variant databases, our proposed method improves the face verification accuracy as compared to subspace and local descriptor methods.

The effectiveness of the proposed approach is depicted in terms of ROC curve shown in Figs. 7(a) and 8(a) for ORL and Essex Grimace database respectively. It is clearly evident from ROC curve that our proposed approach is capable of capturing inter-class and intra-class variations and thus excels as compared to existing state-of-the-art methods.

(b) Non-adaptive multiresolution analysis methods

We additionally compared the proposed approach with some famous non-adaptive multiresolution analysis face recognition methods such as GFC, curvelets, ridgelets, contourlet, and LGBP. It is observed from the experimental results as depicted in Table 4 that the proposed approach yields superior performance against existing MRA based face recognition methods for expression variant face images. The length of the feature vector is presented in Table 4 before performing dimensionality reduction.

Table 3 Performance comparison with subspace and local descriptor methods for ORL and Essex Grimace database

Method	ORL			Essex Grimace		
	FAR (%)	FRR (%)	Verification accuracy (%)	FAR (%)	FRR (%)	Verification accuracy (%)
PCA Turk and Pentland (1991)	0.51	3.77	97.86	6.16	12.33	90.76
KPCA Liu (2004)	0.42	3.46	98.06	2.56	5.19	94.82
WPCA Feng et al. (2000)	0.16	1.91	98.97	7.22	10.39	96.13
LDA Belhumeur et al. (1997)	0.69	1.86	98.72	1.59	7.69	91.20
WLDA Chien and Wu (2002)	0.51	1.32	99.08	0.29	2.14	98.78
KLDA Juwei et al. (2003)	0.26	1.82	98.96	0.95	2.49	98.28
LBP Ahonen et al. (2006)	0.38	0.91	99.35	1.23	3.31	97.73
WLD Li et al. (2013)	0.63	1.18	99.09	0.29	1.46	99.12
Proposed approach	0.38	0.46	99.58	0.29	1.09	99.32

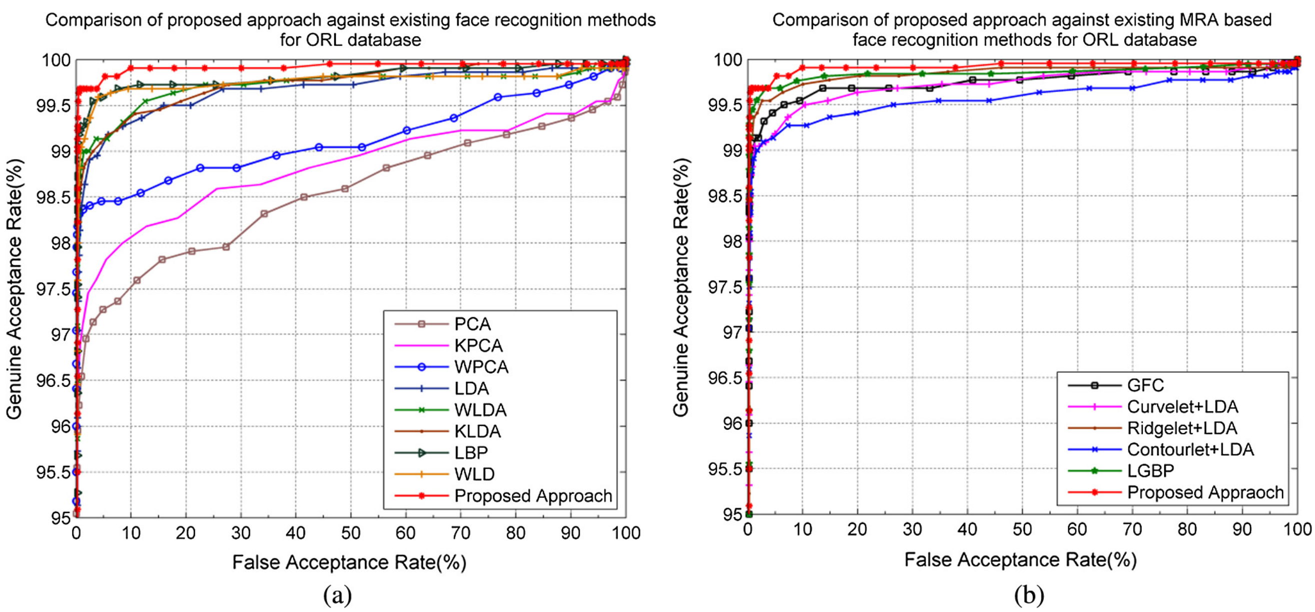


Figure 7 ROC curves for ORL database (a) Subspace and local descriptor methods. (b) Non-adaptive multiresolution analysis methods.

GFC is computationally complicated owing to the convolution of each face image with several Gabor wavelet kernels at many scales and directions (Chien and Wu, 2002). Mandal et al. (2009) required two subbands of fast discrete curvelet transform to form the feature vector for ORL, and Essex Grimace database, however, in our proposed approach we considered only the top level low-frequency *LL* subband for ORL, and Essex Grimace databases. Additionally, the LDA not only provides the dimensionality reduction but also provides the discriminant features from *LL* subband. All these MRA face recognition methods (GFC, curvelet, ridgelet, contourlet, and LGBP) require multiple orientations and scales to describe the face feature vectors effectively (El Aroussi et al., 2011). They have a weakness in terms of high computational costs due to consideration of the large size of feature vectors for achieving a better performance. Moreover, these methods do not offer adaptation in selecting a direction along a sub-block of pixels. The LGBP method provides close verification accuracy to our proposed method, but the size of the feature vector is as high as 151,040. 2-D ADWT locally adapts the filtering directions given in (5) and (6) to face features oriented along edges. Adaptation of filtering directions along such features can be achieved by aligning the direction of wavelet transform with the direction of these features. In addition lifting structure also guarantees perfect reconstruction and the generated multiresolution decomposition image is also similar to that of 2-D DWT. The proposed approach achieves utmost verification accuracy with reduced feature vector size as compared with other multiresolution analysis methods. This indicates the robustness of the proposed approach against expression variation, as depicted in ROC curves as shown in Fig. 7(b) for ORL database and in Fig. 8(b) for Essex Grimace database.

3.4.2. Testing the performance of the methods in illumination variations

The Yale database is considered to investigate the effectiveness of the proposed method for illumination and expression variation on face images. For the Yale face database the genuine attempts are 990 and the impostor attempts are 25,575. Illumination variation in face images increases intra-class variations i.e. FRR significantly and the face recognition method reacts

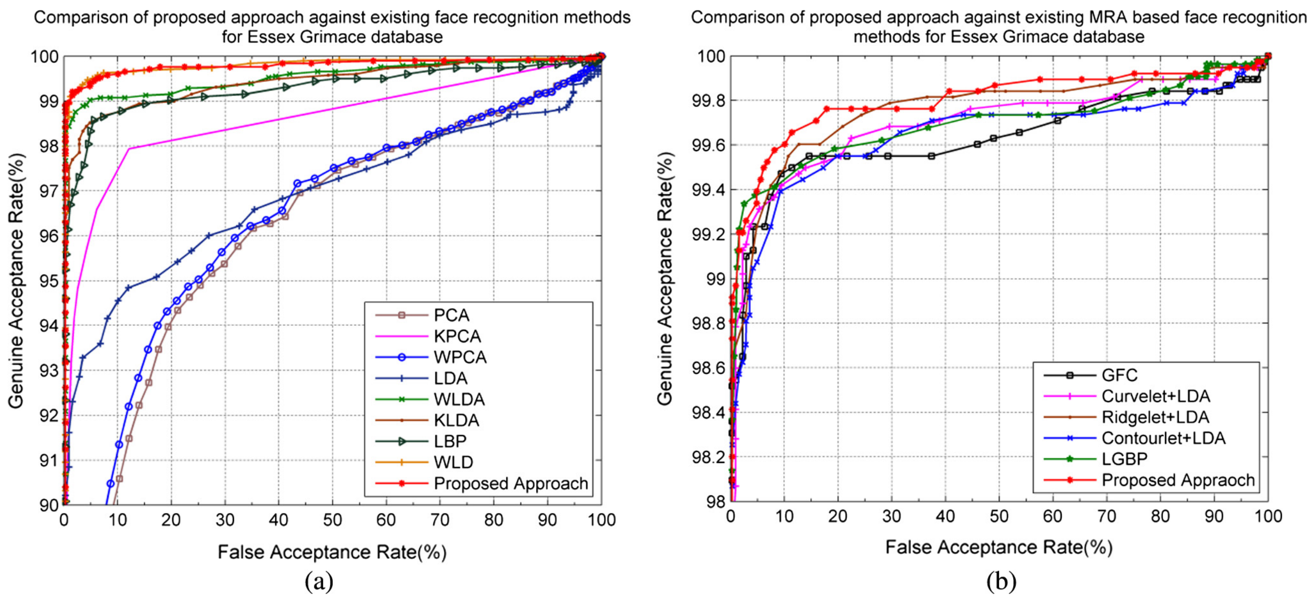


Figure 8 ROC curves for Essex Grimace database (a) Subspace and local descriptor methods. (b) Non-adaptive multiresolution analysis methods.

Table 4 Performance comparison with different multiresolution analysis methods for ORL and Essex Grimace databases

Method	ORL				Essex Grimace			
	FAR (%)	FRR (%)	Verification accuracy (%)	Length of feature vector	FAR (%)	FRR (%)	Verification accuracy (%)	Length of feature vector
GFC Liu and Wechsler (2002)	0.54	1.09	99.18	10,240	0.29	1.48	99.11	10,240
Curvelet + LDA Mandal et al. (2009)	0.62	1.27	99.05	5079	0.95	1.22	98.91	12,729
Ridgelet + LDA Kautkar et al. (2012)	0.58	0.86	99.28	18,432	0.29	1.35	99.18	75,536
Contourlet + LDA Boukabou and Bouridane (2008)	0.77	1.23	99.00	5079	0.29	1.75	98.98	8192
LGBP Zhang et al. (2005)	0.31	0.71	99.48	151,040	0.38	1.63	98.98	151,040
Proposed approach	0.38	0.46	99.58	1024	0.29	1.08	99.32	4096

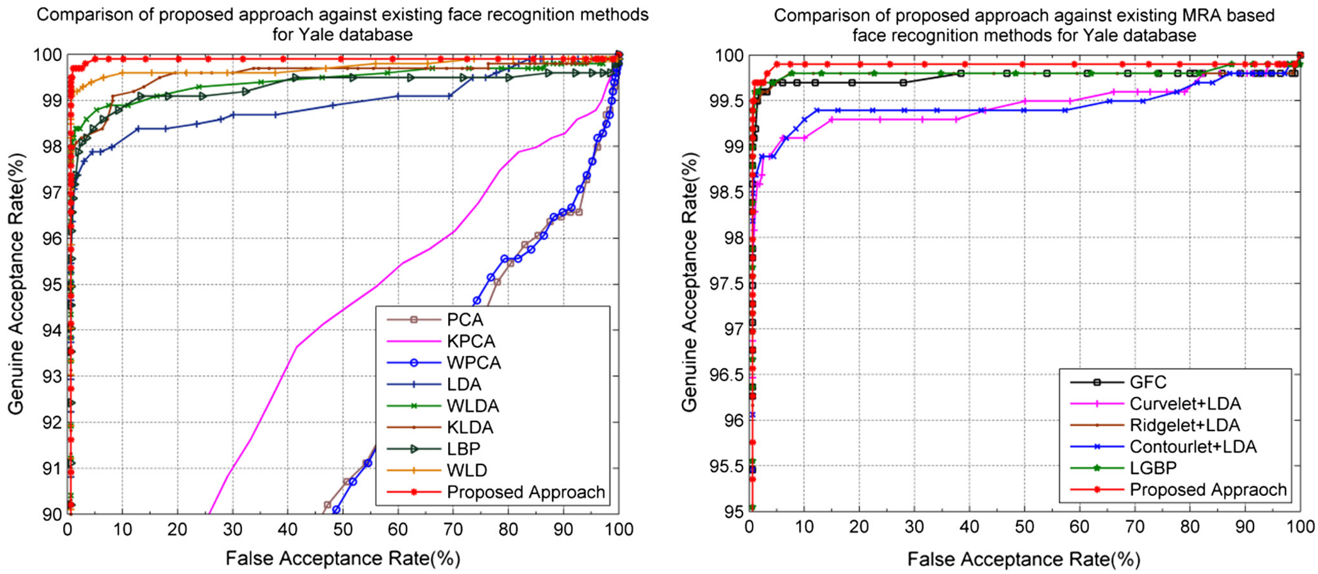
Table 5 Performance comparison with subspace and local descriptor methods for Yale database.

Method	FAR (%)	FRR (%)	Verification accuracy (%)
PCA Turk and Pentland (1991)	6.94	27.17	82.95
KPCA Liu (2004)	4.25	23.94	85.91
WPCA Feng et al. (2000)	10.15	24.65	82.60
LDA Belhumeur et al. (1997)	1.26	2.93	97.91
WLDA Chien and Wu (2002)	0.82	1.92	98.63
KLDA Juwei et al. (2003)	0.65	2.22	98.57
LBP Ahonen et al. (2006)	1.25	2.83	97.96
WLD Li et al. (2013)	0.68	0.91	99.21
Proposed approach	0.68	0.50	99.41

with this to decrease the inter-class variations i.e. FAR. Comparative results for this database are illustrated in Table 5 and in Fig. 9(a) for subspace methods. Performance comparison for non-adaptive MRA methods is illustrated in Table 6 and in Fig. 9(b).

(a) Subspace and local descriptor methods

Considering the number of face images in this database, the maximum length of the feature vector after dimensionality reduction for PCA, KPCA, and WPCA is 164, while for



(a) (b)

Figure 9 ROC curves for Yale database (a) Subspace and local descriptor methods. (b) Non-adaptive multiresolution analysis methods.

Table 6 Performance comparison with different multiresolution analysis methods for Yale database.

Method	FAR (%)	FRR (%)	Verification accuracy (%)	Length of feature vector
GFC Liu and Wechsler (2002)	0.65	1.01	90.17	10,240
Curvelet + LDA Mandal et al. (2009)	1.07	1.72	98.61	5079
Ridgelet + LDA Kautkar et al. (2012)	0.71	0.91	99.19	18,432
Contourlet + LDA Boukabou and Bouridane (2008)	0.89	1.31	98.90	5079
LGBP Zhang et al. (2005)	0.65	0.61	99.37	151,040
Proposed approach	0.68	0.51	99.41	1024

LDA, KLDA, and WLDA the maximum feature vector length is 14. For LBP-based face recognition method, similar setting is applied. The PCA, KPCA, and WPCA methods do not give good results for illumination variant face images, as these methods represent faces with their principal components, but there exists a large variation between the images of the same person due to illumination which severely affects the principal components. The illumination variations mainly affect the high-frequency component of the image and could be detected in the detail high-frequency subbands of the conventional wavelet transform. The conventional 2-D DWT only considers the horizontal and vertical directions and consequently fails to approximate edges which are neither horizontal nor vertical. 2-D ADWT effectively captures the directional edge features from face images due to the nine different candidate directions. These directions and the accompanied 2-D ADWT structure are adequate to reap all the essential illumination and expression relevant information from the face images. The proposed method outperforms subspace and local descriptor methods with the highest verification accuracy of 99.41%. The second highest verification accuracy of 99.20% is obtained by WLD.

(b) Non-adaptive multiresolution analysis methods

The illumination variations mainly affect the high-frequency component of the image and could be detected in the detail high-frequency subbands of the conventional wavelet transform. The similar is factual for curvelets, ridgelets, and contourlets where additional detail high-frequency subband may be considered for capturing illumination variant features. However, the accumulation of the detail high-frequency subbands along with the low-frequency subband increases the feature vector size and the computational time. The nine directions as given in (5) and (6) for the first and second-dimensional liftings are adequate to reap all the essential illumination and expression relevant information from the face images. The consideration of farther away integer samples is very beneficial in extracting the sharp image details and provides energy compaction in the low-frequency subband and preserves the local details of illumination variant features. Hence, even for illumination variant face images our proposed method only considers the low-frequency subband with reduced feature vector length and still manages to give the best results compared to state-of-the-art non-adaptive multiresolution analysis methods as depicted in Table 6 and Fig. 9(b).

3.4.3. Testing the performance of the methods for pose variations

Sterling face database is considered to examine the efficacy of the proposed method for pose variation in face images. For the Sterling face database, the genuine attempts are 1485 and the impostor attempts are 85,833. Due to the pose variations and facial expressions, the intra-class variations may be greater than the inter-class variations. Comparative results for this

Table 7 Performance comparison with subspace and local descriptor methods for Sterling face database.

Method	FAR (%)	FRR (%)	Verification accuracy (%)
PCATurk and Pentland (1991)	11.33	40.47	74.10
KPCALiu (2004)	7.97	42.02	75.00
WPCAFeng et al. (2000)	10.28	41.41	74.16
LDA Belhumeur et al. (1997)	1.22	3.43	97.68
WLDAChien and Wu (2002)	0.49	3.57	97.96
KLDAJuwei et al. (2003)	0.55	1.48	98.98
LBP Ahonen et al. (2006)	0.75	2.15	98.55
WLD Li et al. (2013)	1.18	2.76	98.03
Proposed approach	0.67	1.21	99.06

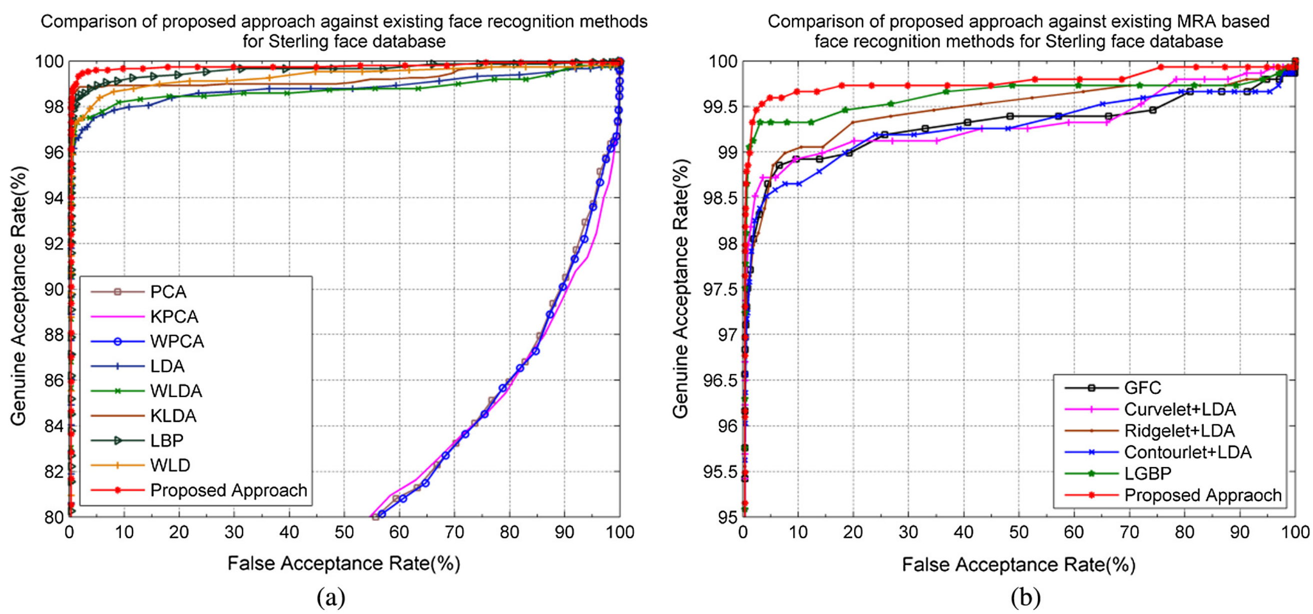


Figure 10 ROC curves for Sterling face database (a) Subspace and local descriptor methods. (b) Non-adaptive multiresolution analysis methods.

database are illustrated in Table 7 and in Fig. 10(a) for subspace methods. Performance comparison for non-adaptive MRA methods is illustrated in Table 8 and in Fig. 10(b).

(a) Subspace and local descriptor methods

Considering the number of face images in this database, the maximum length of the feature vector after dimensionality reduction for PCA, KPCA, and WPCA is 296, while for LDA, KLDA, and WLDA the maximum feature vector length is 32. The PCA, KPCA, and WPCA methods do not give good results for head pose variant face images, as different face images with the same pose may also have appearance dissimilarity due to expression. We can see greater inter-class and intra-class variations in PCA, KPCA, and WPCA methods. Performance improvement could be noticed in LDA, KLDA, WLDA, LBP and WLD, but still less compared to our proposed approach. The proposed method outperforms subspace and local descriptor methods with the highest verification accuracy of 99.06%. The second highest verification accuracy of 98.98% is obtained with KLDA.

(b) Non-adaptive multiresolution analysis methods

In Sterling face database every subject has a different expression and pose condition. GFC, curvelets, contourlets, ridgelets, and LGBP irrespective of providing multiresolution features lack the adaptive directional selectivity which is the key advantage of the proposed approach. Pose variant discontinuities correspond mainly to the high-frequency component of the image. To capture pose variant features more subbands need to be considered to form the feature vector, however, the size of the feature vector and consequently the computational complexity will be increased. The nine directions given in (5) and (6) for the first and second-dimensional liftings are adequate to reap all the essential expression and pose relevant information from the face images. The consideration of farther away integer samples is very beneficial in extracting the sharp image details and provides energy compaction in the low-frequency subband and preserves the local details of expression and poses variant features. Thus the performance verification results support the fact that the proposed approach can overcome the pose variation difficulties in face recognition. Hence, even for pose variant face images our proposed approach only

Table 8 Performance comparison with different multiresolution analysis methods for Sterling face database.

Method	FAR (%)	FRR (%)	Verification accuracy (%)	Length of feature vector
GFC Liu and Wechsler (2002)	0.66	2.69	98.32	10,240
Curvelet + LDA Mandal et al. (2009)	0.95	2.02	98.52	12,729
Ridgelet + LDA Kautkar et al. (2012)	0.61	2.63	98.38	75,536
Contourlet + LDA Boukabou and Bouridane (2008)	0.71	2.83	98.23	8192
LGBP Zhang et al. (2005)	1.16	0.94	98.95	151,040
Proposed approach	0.67	1.21	99.06	4096

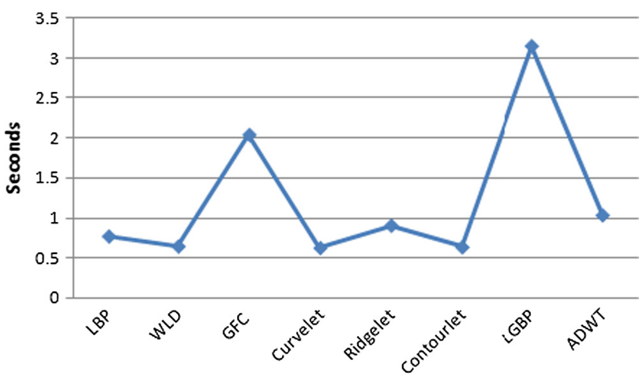


Figure 11 Computational time for ORL face image (128 × 128).

considers the low-frequency subband with reduced feature vector length and still manages to give the best results compared to state-of-the-art non-adaptive multiresolution analysis methods.

3.5. Computational complexity

The proposed method also has a comparable computational time for feature extraction compared with different methods. Fig. 11 shows the computation time to process the ORL database face image of size 128 × 128 pixel resolution and to produce the features. The LGBP method has the highest computation time.

4. Conclusion

In this paper, a new face recognition approach is introduced using 2-D adaptive directional wavelet transform (2-D ADWT) and LDA subspace method. The 2-D ADWT is explored here for the first time for face recognition problem. In the proposed approach, 2-D ADWT performs an adaptive selection of best lifting direction and utilizes integer pixels to perform prediction and update step according to the local characteristics between the pixels. In this work, we selected nine directions with eight vanishing moments for both prediction and update filters. A sub-block partitioning method is applied to locate the sub-block with the optimal direction where the lifting wavelet transform is performed. Experimental results are investigated on ORL, Essex Grimace, Yale, and Sterling face databases. The proposed approach significantly improves the verification performance in comparison with state-of-the-art methods which includes subspace, local descriptor, and non-adaptive multiresolution methods. Performance results in terms of FAR, FRR, and verification accuracy evidently illustrate the superiority of the proposed approach even for appearance variations due to expression, illumination, and pose.

References

Ahonen, T., Hadid, A., Pietikainen, M., 2006. Face description with local binary patterns: application to face recognition. IEEE Trans. Pattern Anal. Mach. Intell. 28, 2037–2041.

Bartlett, M.S., Lades, H.M., Sejnowskim, T.J., 1998. Independent component representations for face recognition, Proceedings of the SPIE, vol. 3299, pp. 528–539.

Belhumeur, N., Hespanha, J.P., Kriegman, D.J., 1997. Eigenfaces vs. fisherfaces: recognition using class specific linear projection. IEEE Trans. Pattern Anal. Mach. Intell. 19 (7), 711–720.

Boukabou, W.R., Bouridane, A., 2008. Contourlet-based feature extraction with PCA for face recognition, in: Proceedings of the 2008 NASA/ESA Conference on Adaptive Hardware and Systems Table of Contents, Washington, DC, USA, pp. 482–486.

Candes, E.J., Demanet, L., Donoho, D.L., Ying, L., 2007. Fast discrete curvelet transform, in: SIAM Multiscale Modeling and Simulations.

Cao, Feilong, Heping, Hu, Jing, Lu, Zhao, Jianwei, Zhou, Zhenghua, Jiao, Wu, 2016. Pose and illumination variable face recognition via sparse representation and illumination dictionary. Knowl.-Based Syst. 107, 117–427.

Chang, C.-L., Girod, B., 2007. Direction-adaptive discrete wavelet transform for image compression. IEEE Trans. Image Process. 16 (5), 1289–1302.

Chen, J., Shan, S., He, C., et al, 2010. WLD: a robust local image descriptor. IEEE Trans. Pattern Anal. Mach. Intell. 32 (9), 1705–1720.

Chien, J.T., Wu, C.C., 2002. Discriminant waveletfaces and nearest feature classifiers for face recognition. IEEE Trans. Pattern Anal. Mach. Intell. 24 (12), 1644–1649.

Daubechies, I., Sweldens, W., 1998. Factoring wavelet transforms in to lifting steps. J. Fourier Anal. Appl. 4 (3), 245–267.

Ding, W., Wu, F., Wu, X., Li, S., Li, H., 2007. Adaptive directional lifting-based wavelet transform for image coding. IEEE Trans. Image Process. 16 (2), 416–427.

Do, M.N., Vetterli, M., 2003. The finite ridgelet transform for image representation. IEEE Trans. Image Process. 12 (1), 16–28.

Do, M.N., Vetterli, M., 2005. The contourlet transform: an efficient directional multiresolution image representation. IEEE Trans. Image Process. 14 (12), 2091–2106.

El Aroussi, M., El Hassouni, M., Ghouzali, S., Rziza, M., Aboutajdine, D., 2011. Local appearance based face recognition method using block based steerable pyramid transform. Signal Process. 91 (1), 38–50.

Feng, G.C., Yuen, P.C., Dai, D.Q., 2000. Human face recognition using PCA on wavelet subband. J. Electron. Imaging 9 (2), 226–233.

Huang, Z.H., Li, W.J., Wang, J., Zhang, T., 2015a. Face recognition based on pixel-level and feature-level fusion of the top-level's wavelet subbands. Inform. Fusion 22, 95–104.

Huang, Z.H., Li, W.J., Shang, J., Wang, J., Zhang, T., 2015b. Non-uniform patch based face recognition via 2D-DWT. Image Vis. Comput. 37, 12–19.

Jadhav, D.V., Holambe, R.S., 2008. Feature extraction using Radon and Wavelet transforms with application to face recognition. J. Neurocomputing 72, 1951–1959.

Jain, A.K., Ross, A., Prabhakar, S., 2006. Biometrics: a tool for information security. IEEE Trans. Inf. Forensics Secur. 1 (2), 125–143.

Jin, A.T.B., Ling, D.N.C., Ong, T.S., 2004. An efficient fingerprint verification system using integrated wavelet and Fourier-Mellin invariant transform. Image Vision Comput. 22 (6), 503–513.

Jonsson, K., Kittler, J., Li, Y.P., Matas, J., 2002. Support vector machines for face authentication. Image Vis. Comput. 20 (5–6), 369–375.

Juwei, L., Plataniotis, K.N., Venetsanopoulos, A.N., 2003. Face recognition using kernel direct discriminant analysis methods. IEEE Trans. Neural Networks 14, 117–126.

Kautkar, S.N., Atkinson, G.A., Smith, M.L., 2012. Face recognition in 2D and 2.5D using ridgelets and Photometric stereo, Pattern Recognition, vol. 45, pp. 3317–3327.

Kovacevic, J., Sweldens, W., 2000. Wavelet families of increasing order in arbitrary dimensions. IEEE Trans. Image Process. 9 (3), 480–496.

Lei, Ying-Ke, Han, Hui, Hao, Xiaojun, 2015. Discriminant sparse local spline embedding with application to face recognition. Knowl.-Based Syst. 89, 47–55.

Li, S., Gong, D., Yuan, Y., 2013. Face recognition using weber local descriptors. Neurocomputing 122, 272–278.

Liu, C., 2004. Gabor-based kernel PCA with fractional power polynomial models for face recognition. IEEE Trans. Pattern Anal. Mach. Intell. 26 (5), 572–581.

Liu, C.J., Wechsler, H., 2002. Gabor feature based classification using the enhanced fisher linear discriminant model for face recognition. IEEE Trans. Image Process. 11 (4), 467–476.

Maleki, A., Rajaei, B., Pourreza, H.R., 2012. Rate-distortion analysis of directional wavelets. IEEE Trans. Image Process. 21 (2), 588–600.

Mandal, T., Wu, Q.M.J., Yuan, Y., 2009. Curvelet based face recognition via dimension reduction. Signal Process. 89, 2345–2353.

Muqeet, Mohd Abdul, Holambe, Raghunath S., 2016. Enhancing face recognition performance using triplet half band wavelet filter bank. Int. J. Image Graph. Signal Process. 8 (12), 62–70. http://dx.doi.org/10.5815/ijigsp.2016.12.08.

Muqeet, M.A., Holambe, R.S., 2016. Face recognition using LDA based generalized half band polynomial wavelet filter bank, 2016 International Conference on Electrical, Electronics, and Optimization Techniques (ICEEOT), Chennai, India, pp. 4649–4653. doi: 10.1109/ICEEOT.2016.7755601.

[Online]. Available: <http://cvc.yale.edu/projects/yalefaces/yalefaces.html>.

[Online]. Available: <http://www.uk.research.att.com/pub/data/att_faces.zip>.

[Online]. Available: <http://cswww.essex.ac.uk/mv/allfaces/grimace.zip>.

Schoelkopf, B., Smola, A.J., Mueller, K.R., 1997. Kernel principal component analysis, in: Lecture Notes in Computer Science.

Simoncelli, E.P., 1996. A rotation-invariant pattern signature, in: Third IEEE International Conference on Image Processing, Lausanne Switzerland, vol. 3, pp. 185–188.

Struc, V., Pavesic, N., 2009. Gabor-based kernel partial-least-squares discrimination features for face recognition. Informatica 20 (1), 115–138.

Sweldens, W., 1998. The lifting scheme: construction of second generation wavelets. SIAM J. Math. Anal. 29 (2), 511–546.

Tang, Min., Chen, Feng., 2013. Facial expression recognition and its application based on curvelet transform and PSO-SVM. Optik 14, 5401–5406.

Turk, M., Pentland, A., 1991. Eigenfaces for recognition. J. Cognit. Neurosci. 3 (1), 71–86.

The University of Stirling online database <http://pics.psych.stir.ac.uk/>.

Vinay, A., Shekhar, V.S., Murthy, K.B., Natarajan, S., 2015. Face recognition using gabor wavelet features with PCA and KPCA-A comparative study. Procedia Comput. Sci. 57, 650–659.

Zhang, W., Shan, S., Gao, W., Chen, X., Zhang, H., 2005. Local Gabor binary pattern histogram sequence (LGBPHS): a novel non-statistical model for face representation and recognition, in: Tenth IEEE International Conference on Computer Vision (ICCV'05), Beijing, China, 2005, pp. 786–791.