



# A TLBO based gradient descent learning-functional link higher order ANN: An efficient model for learning from non-linear data



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Received 24 May 2015; revised 11 December 2015; accepted 6 January 2016  
Available online 28 March 2016

## KEYWORDS

Teaching–learning based optimization;  
Functional link artificial neural network;  
Gradient descent learning;  
Classification;  
Data mining

**Abstract** All the higher order ANNs (HONNs) including functional link ANN (FLANN) are sensitive to random initialization of weight and rely on the learning algorithms adopted. Although a selection of efficient learning algorithms for HONNs helps to improve the performance, on the other hand, initialization of weights with optimized weights rather than random weights also play important roles on its efficiency. In this paper, the problem solving approach of the teaching learning based optimization (TLBO) along with learning ability of the gradient descent learning (GDL) is used to obtain the optimal set of weight of FLANN learning model. TLBO does not require any specific parameters rather it requires only some of the common independent parameters like number of populations, number of iterations and stopping criteria, thereby eliminating the intricacy in selection of algorithmic parameters for adjusting the set of weights of FLANN model. The proposed TLBO-FLANN is implemented in MATLAB and compared with GA-FLANN, PSO-FLANN and HS-FLANN. The TLBO-FLANN is tested on various 5-fold cross validated benchmark data sets from UCI machine learning repository and analyzed under the null-hypothesis by using Friedman test, Holm's procedure and post hoc ANOVA statistical analysis (Tukey test & Dunnett test). © 2016 King Saud University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

## 1. Introduction

Classification and forecasting tasks are typical process of learning and knowledge discovery from data in the field of data mining. Many applications of classification tasks have been reported in recent years from emerging areas of science and engineering (Yang et al., 2013; Hajmohammadi et al., 2014; He et al., 2014; Uysal and Gunal, 2014; Tolambiya et al., 2010; Mei et al., 2014; Kianmehr et al., 2010; Gillies

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**Table 1** Previous literatures on teaching learning based optimization (TLBO).

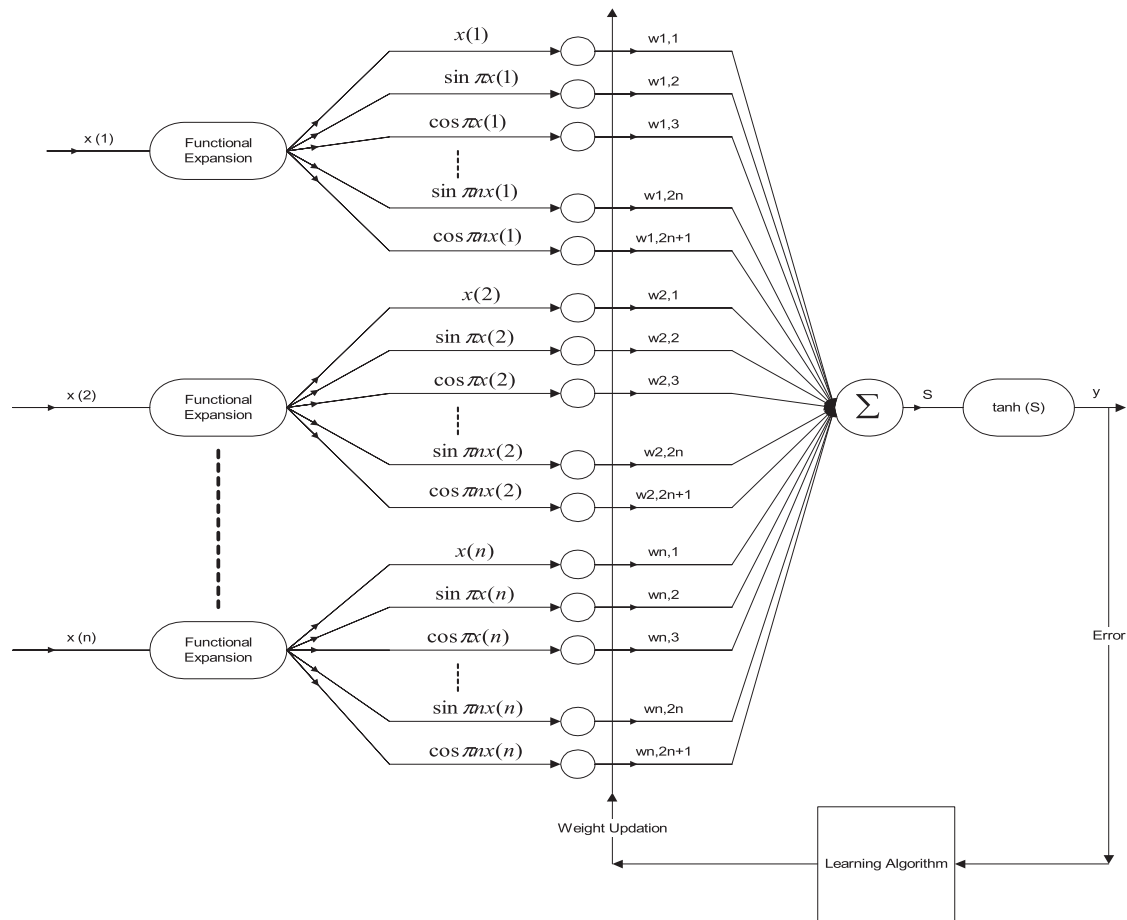
Reference	Contribution	Area of application	Year
Toğan (2012)	TLBO	Engineering design	2012
Niknam et al. (2012)	MOTLBO	Economic dispatch	2012
Zou et al. (2013)	TLBO	Multi-objective optimization	2013
Roy et al. (2013)	QOTLBO	Hydro thermal scheduling	2013
Mandal and Roy (2013)	QOTLBO	Power dispatch	2013
García and Mena (2013)	MTLBO	Distributed generation	2013
Venkata Rao and Kalyankar (2013)	TLBO	Machining processes	2013
Roy (2013)	TLBO	Scheduling problem	2013
Roy and Bhui (2013)	QOTLBO	Load dispatch	2013
Singh et al. (2013)	TLBO	Power system	2013
Satapathy et al. (2013a)	WTLBO	Optimization	2013
Satapathy et al. (2013b)	OTLBO	Optimization	2013
Tuo et al. (2013)	HSTL	Optimization	2013
Theja et al. (2013)	TLBO	Power system	2013
Sultana and Roy (2014)	TLBO	Optimal capacitor placement	2014
Azizipanah-Abarghooee et al. (2014)	Gradient based modified TLBO with Black hole	Scheduling of thermal power systems	2014
Arya and Koshti (2014)	TLBO	Load shedding	2014
Khalghani and Khoob (2014)	TLBO	Power quality	2014
Niu et al. (2014)	STLBO	Fuel and solar cell models	2014
Moghadam and Seifi (2014)	Fuzzy-TLBO	Energy loss minimization	2014
Cheng (2014)	TLBO	Temperature calculations	2014
Barisal (2015)	TLBO	Load frequency control	2015
Ghasemi et al. (2015a)	GBTLBO	Power dispatch problem	2015
Chen et al. (2015)	ITLBO	Optimization	2015
Sahu et al. (2016)	TLBO	Power system	2015
Ghasemi et al. (2015b)	ITLBO	Power flow	2015

et al., 2013; Lai and Garibaldi, 2013), which has given motivation and direction to the application of the classification task in data mining. Although a good number of traditional classification methods are proposed by many researchers (Quinlan, 1993; Yung and Shaw, 1995; Hamamoto et al., 1997; Yager, 2006), for the first time, Zhang (2000) realized that artificial neural network (ANN) models are alternatives to various conventional classification methods which are based on statistics. ANNs are capable of generating complex mapping between input and the output space and thus they can form arbitrarily complex nonlinear decision boundaries. Along the way, there are already several artificial neural networks, each utilizing a different form of learning or hybridization. As compared to higher order neural network, classical neural networks (Example: MLP) are suffering from slow convergence and unable to automatically decide the optimal model of prediction for classification. In the last few years, to overcome the limitations of conventional ANNs, some researchers have focused on higher order neural network (HONN) models (Redding et al., 1993; Goel et al., 2006) for better performance.

In this paper, it is an attempt to design HONN model with competitive learning based on a new meta-heuristic optimization algorithm for classification of benchmark data sets considered from well known machine learning data repository. The encouraging results and absence of the typical algorithm dependent parameters of the recently developed TLBO algorithm (Rao et al., 2011) inspired us to develop the model: TLBO-FLANN: a hybrid TLBO based gradient descent learning-functional link higher order ANN (FLANN) model for learning from non-linear data.

## 2. Literature survey

A good number of research papers is reported on various FLANN models and its application in classification, prediction and forecasting in recent years. Also excellent efforts are made to improve the performance of these FLANN models by using various optimization techniques. This section presents various previous works reported on FLANN models in classification, prediction and forecasting in data mining. A classification method by using FLANN with least complex architecture as compared to multilayer perceptron is anticipated by Mishra and Dehuri (2007) and the proposed method is found to be efficient in terms of ability of handling linearly non-separable classes by increasing the dimension of input space. In most cases, the performance and processing time of FLANN model is found to be better than the other models. A survey on FLANN is made and PSO based back propagation learning is proposed by Dehuri and Cho (2009). The basic concept of FLANN, associated basis functions, learning schemes and development of FLANNs over time are discussed in this paper. Also authors have proposed a Chebyshev-FLANN with hybrid PSO-back propagation learning for classification. The proposed model is proven to be better as compared to FLANN by testing with benchmark data sets. Patra et al. (2009) suggested an efficient FLANN model for making stock price prediction of the closing price of US stocks and the proposed model is compared with MLP-based prediction model through several experiments. This proposed trigonometric FLANN has shown superior performance over MLP by making more accurate predictions of stock prices. Prediction of the causing genes in gene diseases by FLANN



**Figure 1** Functional link higher order artificial neural network architecture.

**Table 2** Various notations and symbols used in algorithmic design of proposed scheme.

Notation used	Meaning
$X$	Population of weight-sets
$x_i$	Each individual weight-set in $X$
$w_{i,j}$	$j$ 'th weight value of $i$ 'th weight-set ( $x_i$ )
RMSE	Root mean square error
$F_{x_i}$	Fitness of $x_i$ ( $i$ 'th weight-set in $X$ )
$X_{\text{mean}}$	Mean of the population $X$
$x_{\text{teacher}}$	Selected weight-set as teacher
$T_F$	Teaching Factor
$X(i)$	$i$ 'th weight-set of the population $X$
$X_{\text{next}}(i)$	$i$ 'th weight-set of the next population $X$
$\Phi$	Functionally expanded input values
$T$	Target values
$\mu$	Gradient descent learning parameter
$Y(k)$	Output of FLANN model for $k$ 'th pattern
$t(k)$	Target value of $k$ 'th pattern
$e(k)$	Error value of $k$ 'th pattern
$\Delta W$	Error term of the network

model is proposed by [Sun et al. \(2009\)](#) and compared with multilayer perceptron (MLP) and support vector machines (SVM). In this study, three classifiers (i.e. MLP, SVM, FLANN) have been implemented and the performance of FLANN classifier is found to be better than MLP and SVM.

[Chakravarty and Dash \(2009\)](#) have proposed functional link neural fuzzy (FLNF) model to predict the stock market indices and compared with FLANN model in terms of root mean square error. The simulation results have been demonstrated to claim better performance of FLNF over FLANN. Also the local minima problem which may arise in back propagation learning algorithm is addressed by using particle swarm optimization (PSO). A FLANN based model with least mean square and recursive least square learning algorithm is employed by [Majhi et al. \(2009\)](#) for forecasting of the stock markets. Between least mean square learning based FLANN and recursive least square learning FLANN, the recursive least square learning FLANN is found to be efficient and has less computational complexity. A compact and accurate hybrid FLANN classifier (HFLNN) has been proposed by [Dehuri and Cho \(2010\)](#) by selecting an optimal subset of favorable input features by eliminating features with fewer or no predictive information and this method is found to be better as compared to FLANN and RBFN. The MLP, FLANN and PSO-FLANN classification models are used and tested by [Mishra et al. \(2012\)](#) for classification of biomedical data. In this paper, an efficient dynamic classifier fusion (DCF) has been proposed along with principal component analysis (PCA) scheme which is used to extract important input features. Extracted features are then supplied to LMS classifiers, with learning based on back propagation and PSO algorithm. Although MLP is a traditional ANN, surprisingly, in this

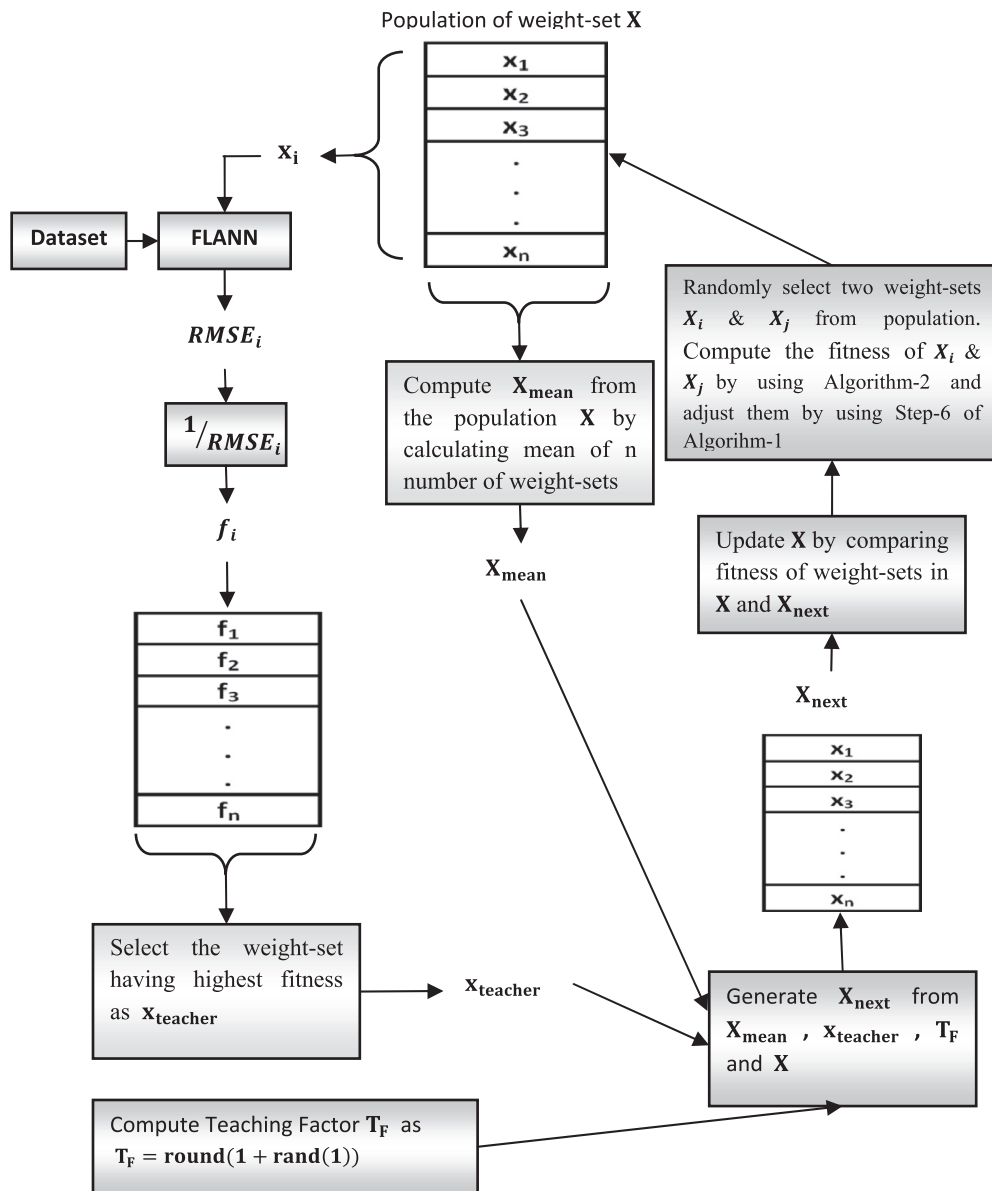


Figure 2 Working procedure of the proposed scheme.

study, they found MLP is better as compared to FLANN and PSO-FLANN. Dehuri et al. (2012) have proposed an IPSO (improved PSO) based FLANN classifier (IPSO-FLANN) and compared with MLP, support vector machine (SVM), RBFN, FLANN with gradient descent learning and fuzzy swarm net (FSN) model. Initially, the set of weight values of FLANN are optimized using IPSO and then those are supplied to FLANN classifier along with functionally expanded (using trigonometric basis functions) input patterns for classification. The ISO-FLANN is simple in architecture and found to be better as compared to MLP, SVM, FLANN with gradient descent learning and FSN. An efficient classification method based on FLANN and a hybrid learning scheme based on PSO and GA have been proposed by Naik et al. (2015a) and it is found to be relatively better in performance as compared to other alternatives. The PSO, GA and the gradient descent search are used iteratively to adjust the parameters of FLANN until the error is less than the required value, which helps the

FLANN model to get better classification accuracy. Naik et al. (2015b) have designed a honey bee mating optimization (HMBO) based learning scheme for FLANN classifier and compared with FLANN, GA based FLANN, PSO based FLANN and HS based FLANN classification model. The proposed method mimics the iterative mating process of honey bees and strategies to select eligible drones for the mating process, selection of best weights for FLANN classifiers. A novel FLANN with a harmony search (HS) algorithm-based GDL method is proposed by Naik et al. (2014), which requires less mathematical computation as compared to other similar methods. The HS based FLANN (HS-FLANN) is more efficiently able to classify the data than PSO-based FLANN and GA based FLANN.

In this paper, a FLANN model with hybrid TLBO based gradient descent learning (GDL) method for learning from non-linear data for classification task has been proposed and compared with previously available alternatives.

**Table 3** Details on data sets used for experiments.

Data sets	Number of pattern	Number of features (excluding class label)	Number of classes
Wine	178	13	03
Iris	150	04	03
Hayesroth	160	04	03
Monk 2	256	06	02
Ionosphere	351	33	02
Hepatitis	80	19	02
Pima	768	08	02
New Thyroid	215	05	03
Bupa	345	06	02
Dermatology	256	34	06
Heart	270	13	02

### 3. Design issues identification

All the FLANN models discussed in the literature survey implement some form of learning methods which learns from past data in forecasting and classification tasks in data mining. Almost all HONNs including FLANN are sensitive to random initialization of weights and rely on the learning algorithm adopted. Although a selection of efficient learning algorithms for HONNs helps to improve the performance, initialization of weights with optimized weights rather than random weights also play important roles in efficiency of HONNs. In the literature survey, it was noticed that, almost all the previously published works have addressed the issue of random initialization of weights in FLANN by using various optimization algorithms like genetic algorithm (GA) (Holland, 1992; Goldberg, 1989), particle swarm optimization (PSO) (Kennedy et al., 1995), harmony search (HS) (Geem et al., 2001), honey-bee mating optimization (HBMO) (Abbass et al., 2001) etc. In these papers, the optimization algorithms (GA, PSO, Improved PSO, HS, HMBO etc.) are used to select the best set of weights for FLANN models for various applications like prediction, classification and forecasting. Although it is reported that these optimization techniques are successfully used in FLANN models for implementation in improved models like GA based FLANN (GA-FLANN) (Dehuri et al., 2012), PSO based FLANN (PSO-FLANN) (Dehuri et al., 2012), IPSO based FLANN (IPSO-FLANN) (Dehuri et al., 2012), HBMO based FLANN (HBMO-FLANN) (Naik et al., 2015b) and HS based FLANN (HS-FLANN) (Naik

et al., 2014), some weakness of these implementations are the requirement of various algorithmic controlling parameters like: (i) selection of mutation rate and type of crossover operator in GA in GA-FLANN, (ii) choosing of appropriate values of inertia weight ( $\lambda$ ), coefficient  $c1$  and  $c2$  in PSO in PSO-FLANN and IPSO-FLANN (iii) types of crossover operator selection and choosing drone and worker ratio in HBMO in HBMO-FLANN and (iv) selection of pitch adjustment rate, harmony memory consideration rate and bandwidth in HS in HS-FLANN. The performances of these models are dependent upon such controlling parameters and any changes in these parameters may not only lead to increase the effort to develop the program but also the time and space complexity of the algorithm increases.

Keeping this in view, a new teaching and learning inspired algorithm (TLBO) is used in FLANN learning model with gradient descent learning (GDL) (Rumelhart et al., 1986) scheme for classification. This paper attempts to address the intricacy in adjusting the set of weights of the FLANN model by using an appropriate learning algorithm. Here the problem solving approach of the TLBO along with learning ability of the GDL is used to obtain the optimal set of weights of FLANN model. In this paper, a FLANN model has been designed with TLBO algorithm which does not require any algorithmic specific parameters, rather it requires some of the common independent parameters like number of populations, number of

**Table 5** Comparison between GA-FLANN, PSO-FLANN, HS-FLANN and TLBO-FLANN models based on the fitness.

Datasets	Fitness obtained by various hybrid models			
	GA-FLANN	PSO-FLANN	HS-FLANN	TLBO-FLANN
Wine	2.462398	2.462398	2.534043	3.057658
Iris	5.972679	5.97268	5.97268	9.411738
Hayesroth	1.888252	1.869618	1.895258	3.118014
Monk 2	2.024735	2.033361	2.037116	3.057657
Ionosphere	1.677268	1.696773	1.67727	2.382388
Hepatitis	2.58331	2.813839	3.276094	2.349537
Pima	2.216361	2.216474	2.217241	2.220777
New Thyroid	1.888252	1.869618	2.675806	3.118014
Bupa	1.54692	1.547116	1.548419	2.162731
Dermatology	1.888967	1.981951	2.267907	3.690652
Heart	2.128121	2.127275	2.128124	2.696475

**Table 4** Data sets prepared in 5-fold cross validation for training and testing phase.

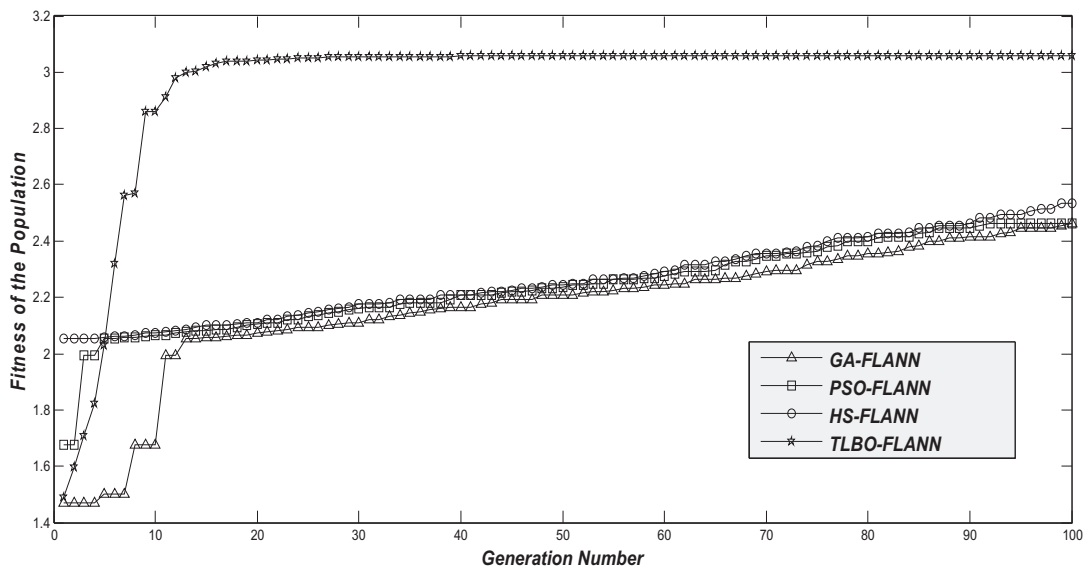
Dataset	Data files	Number of Pattern	Task	Number of Pattern in class-1	Number of Pattern in class-2	Number of Pattern in class-3
New Thyroid	Newthyroid-5-1tra.dat	172	Training	120	28	24
	Newthyroid-5-1tst.dat	43	Testing	30	07	06
	Newthyroid-5-2tra.dat	172	Training	120	28	24
	Newthyroid-5-2tst.dat	43	Testing	30	07	06
	Newthyroid-5-3tra.dat	172	Training	120	28	24
	Newthyroid-5-3tst.dat	43	Testing	30	07	06
	Newthyroid-5-4tra.dat	172	Training	120	28	24
	Newthyroid-5-4tst.dat	43	Testing	30	07	06
	Newthyroid-5-5tra.dat	172	Training	120	28	24
	Newthyroid-5-5tst.dat	43	Testing	30	07	06

**Table 6** Comparison between GA-FLANN, PSO-FLANN, HS-FLANN and TLBO-FLANN models based on average fitness on 11 numbers of data sets.

	Various hybrid models			
	GA-FLANN	PSO-FLANN	HS-FLANN	TLBO-FLANN
Average RMSE on 11 data sets	0.473349	0.468362	0.442128	0.345602

**Table 7** Friedman’s ranks of various models on the data sets based on the maximum fitness obtained.

Datasets	Fitness obtained by various hybrid models			
	GA-FLANN	PSO-FLANN	HS-FLANN	TLBO-FLANN
Wine	2.462398 (3)	2.462398 (3)	2.534043 (2)	3.057658 (1)
Iris	5.972679 (3)	5.97268 (2)	5.97268 (2)	9.411738 (1)
Hayesroth	1.888252 (3)	1.869618 (4)	1.895258 (2)	3.118014 (1)
Monk 2	2.024735 (4)	2.033361 (3)	2.037116 (2)	3.057657 (1)
Ionosphere	1.677268 (4)	1.696773 (2)	1.67727 (3)	2.382388 (1)
Hepatitis	2.58331 (3)	2.813839 (2)	3.276094 (1)	2.349537 (4)
Pima	2.216361 (4)	2.216474 (3)	2.217241 (2)	2.220777 (1)
New Thyroid	1.888252 (3)	1.869618 (4)	2.675806 (2)	3.118014 (1)
Bupa	1.54692 (4)	1.547116 (3)	1.548419 (2)	2.162731 (1)
Dermatology	1.888967 (4)	1.981951 (3)	2.267907 (2)	3.690652 (1)
Heart	2.128121 (3)	2.127284 (4)	2.128124 (2)	2.696475 (1)
Friedman’s rank in average	3.455	3	2	1.272



**Figure 3** Observation on improvements in fitness of population in various iterations on Wine dataset.

iterations and stopping criteria (like other optimization techniques). Starting from the development of TLBO, it has been of keen interest among the diversified researchers and has been used in various real life applications (Vedat Toğan, 2012; Niknam et al., 2012; Zou et al., 2013; Roy et al., 2013; Mandal and Roy, 2013; García and Mena, 2013; Venkata Rao and Kalyankar, 2013; Roy, 2013; Roy and Bhui, 2013; Singh et al., 2013; Satapathy et al., 2013a,b; Tuo et al., 2013; Theja et al., 2013; Sultana and Roy, 2014; Azizipanah-Abarghooee et al., 2014; Arya and Koshti, 2014; Khalghani and Khoob, 2014; Niu et al., 2014; Moghadam and Seifi, 2014; Cheng, 2014; Barisal, 2015; Ghasemi et al., 2015a;

Chen et al., 2015; Sahu et al., 2016; Ghasemi et al., 2015b) (Table 1). Although Črepinšek et al. (2012) had raised some issues relating the performance of TLBO such as: (i) incorrect use of formulae to evaluate various fitness functions,(ii) controlling parameters, (iii) improper experiment settings etc.; later on Waghmare (2013) has correctly reexamined, commented and tested on the above raised issues on TLBO. He has not only tested the algorithm by considering a number of both constrained and unconstrained functions and found the algorithm outperforms the other existing evolutionary algorithms, but he has also been able to prove the misinterpretation and misconception created by Črepinšek

et al. about the understanding point of view of TLBO. Also, he has properly justified the less controlling parameter required to supply for TLBO; in fact only the common controlling parameters are required during the experimental analysis of TLBO, which is claimed by testing the functions and their improved results. However, in this paper, after all the required experimental settings, proper tuning of the common parameters and from the experimental results, it is found that TLBO outperforms some other existing techniques.

The remaining of the paper is organized as follows: Section 4 describes the preliminaries like TLBO algorithm, concept of FLANN and GDL. Section 5 includes the proposed TLBO-FLANN learning model. Section 6 contains experimental setup & data preparation. Section 7 discusses the experimental study and result analysis of the proposed method. Section 8 outlines different statistical tests. Section 9 presents time complexity analysis. Section 10 concludes our work with the future development aspects followed by References.

#### 4. Preliminaries

This section presents basic concepts of TLBO, FLANN and GDL which are the background for the development of the proposed model.

##### 4.1. Teaching learning based optimization

TLBO (Rao et al., 2011) is a recently developed and competent optimization technique which is inspired by natural teaching-learning process in an education system. TLBO mimics the strategy used by a teacher to teach a group of students effectively and captures the effect of teacher on the learning process of the students. The working procedure of TLBO suggested by Rao et al., 2011 can be realized as follows:

##### Teacher phase

**Step-1.** Initialize population of students  $X$  (candidate solutions) randomly.

**Step-2.** Calculate the mean of each student in the population ( $X_{\text{mean}}$ ).

**Step-3.** Compute the fitness of each student in the population and find out the best solution ( $X_{\text{teacher}}$ ).

**Step-4.** Generate a new population by modifying the solutions in initial population based on best solution (teacher), mean of students in the population (mean) and teaching factor  $T_F$ .

**for**  $i = 1:1$ : nos of student in the population  $X$

$T_F = \text{round}(1 + \text{rand}(1))$

$X_i(\text{new}) = X_i(\text{old}) + \text{rand}(1)(X_{\text{teacher}} - T_F * X_{\text{mean}})$

**Endfor**

##### Student phase

**Step-5.** Update population of student  $X$  by comparing fitness of students in old population  $X$  and new population  $X_{\text{new}}$ .

**for**  $i = 1:1$ : nos of student in the population  $X$

**if** ( $X_i(\text{old}) < X_i(\text{new})$ )

$X_i = X_i(\text{new})$

**Else**

$X_i = X_i(\text{old})$

**endif**

**endifor**

**Step-6.** Randomly select two no. of student from the population and improve them.

Select  $i$ th and  $j$ th student  $X_i$  and  $X_j$  randomly from the population.

**If** (fitness of  $X_i < \text{fitness of } X_j$ )

$X_i(\text{new}) = X_i(\text{old}) + \text{rand}(1)(X_j - X_i)$

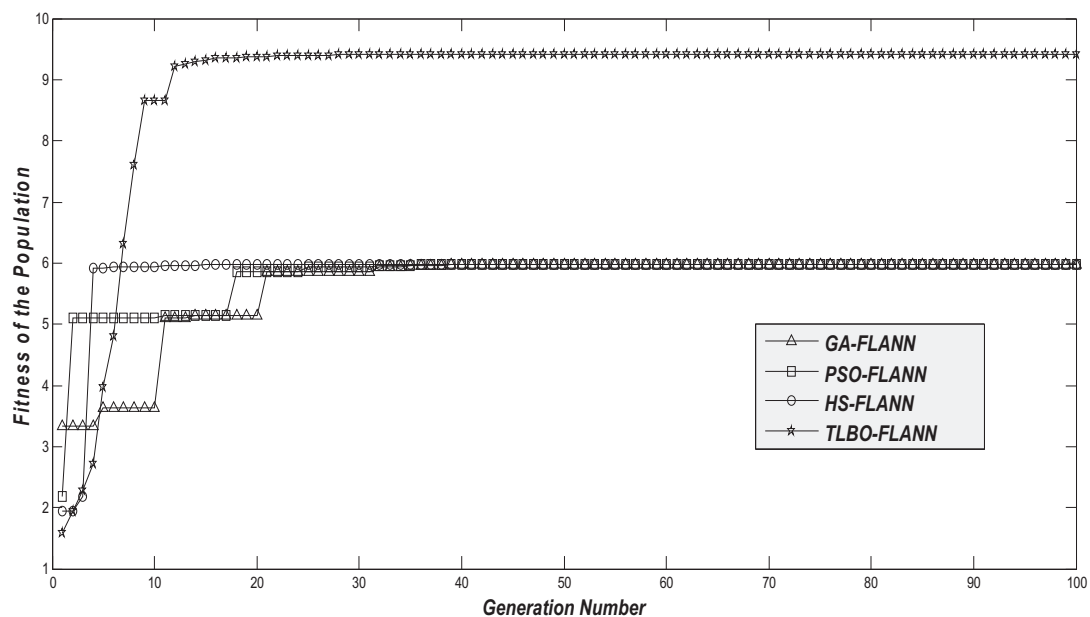
**Else**

$X_j(\text{new}) = X_j(\text{old}) + \text{rand}(1)(X_i - X_j)$

**Ifend**

**Step-7.** Check for termination criteria. If reached stop. Else go to step-2.

**Step-8.** Exit



**Figure 4** Observation on improvements in fitness of population in various iterations on Iris dataset.

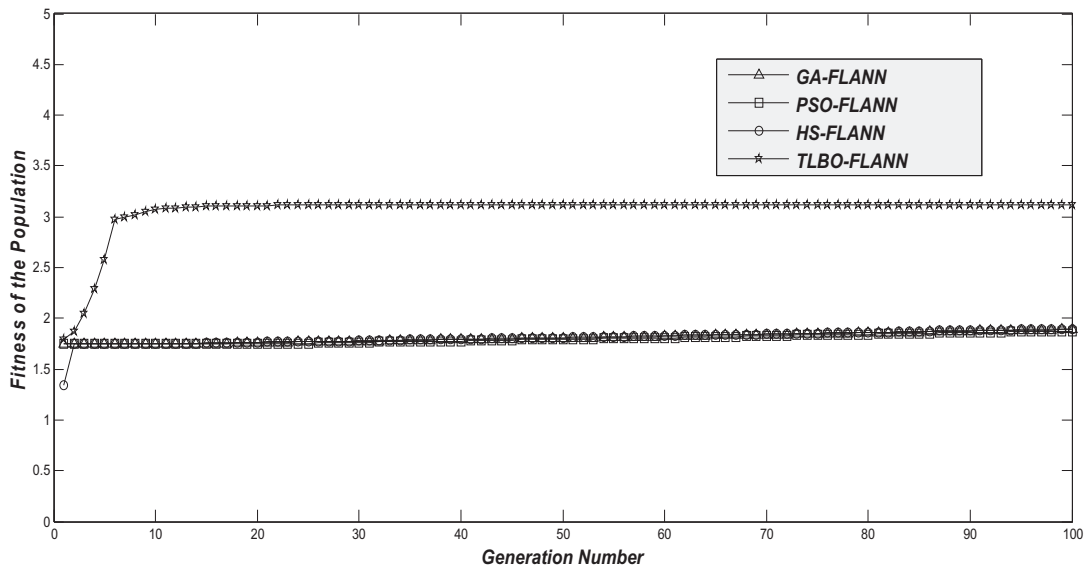


Figure 5 Observation on improvements in fitness of population in various iterations on Hayesroth dataset.

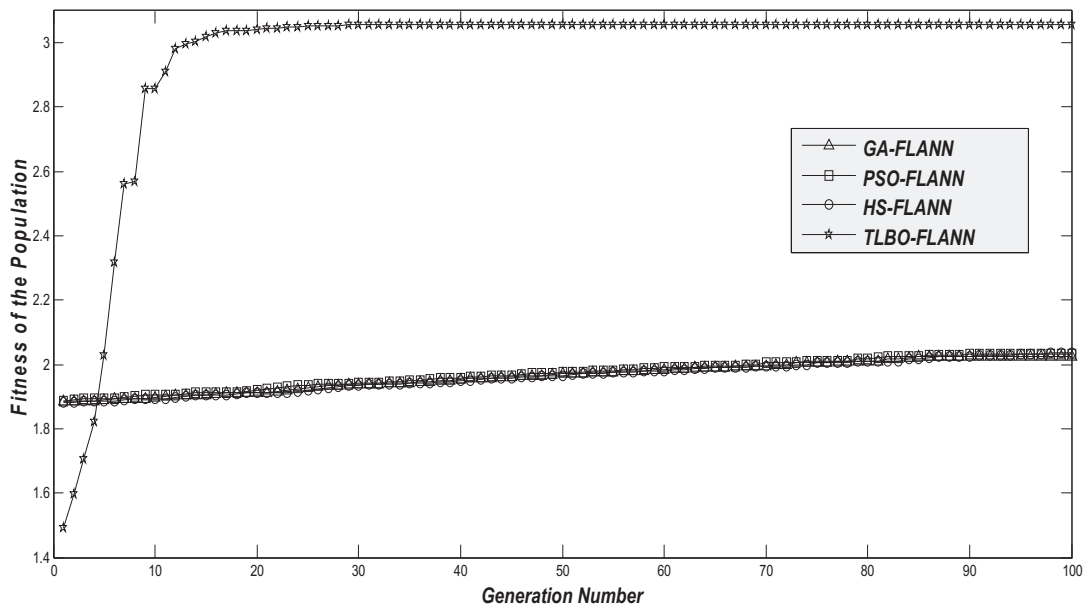


Figure 6 Observation on improvements in fitness of population in various iterations on Monk2 dataset.

4.2. Functional link artificial neural network

FLANN (Pao, 1989; Pao and Takefuji, 1992) is a class of higher order neural network that makes use of a higher combination of its inputs. Even if it has a single-layer network, still it is capable of handling non-linear separable classification tasks as compared to MLP. The error of FLANN is calculated based on net output and given target value. A suitable learning method can be adapted to adjust weight values of FLANN based on error of the network. Fig. 1 depicts the basic architecture of FLANN. The complete concepts of FLANN and its

implementations may be found from recently published related work (Naik et al., 2015a, b, 2014).

4.3. Gradient descent learning

Gradient descent learning (GDL) (Rumelhart et al., 1986) is the most commonly used training method in which weights are changed in such a way that network error is declined as rapidly as possible. The complete implementations of GDL for FLANN can be found from recently published related work (Majhi et al., 2009; Naik et al., 2015a,b, 2014). Basically,



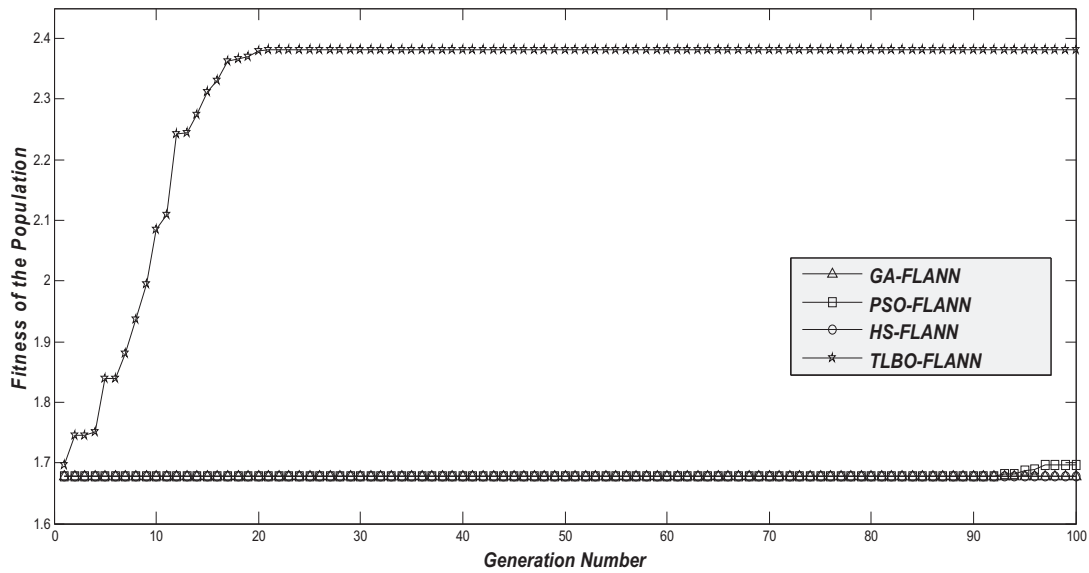


Figure 7 Observation on improvements in fitness of population in various iterations on Ionosphere dataset.

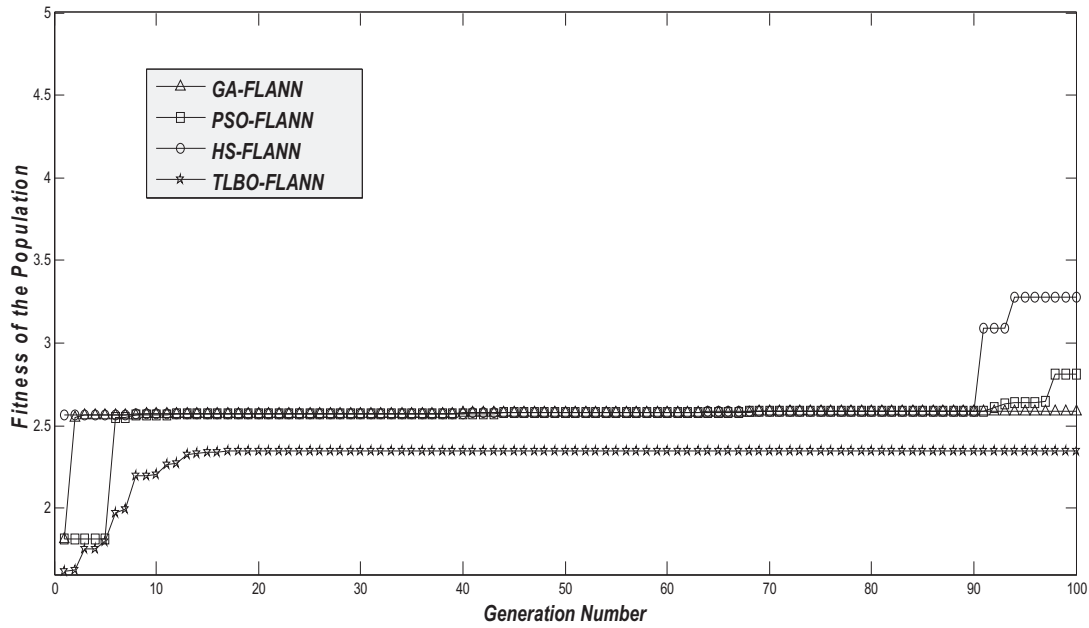


Figure 8 Observation on improvements in fitness of population in various iterations on Hepatitis dataset.

a better learning algorithm helps the ANN models for faster convergence. Further, a use of competitive optimization technique can improve the performance of a learning algorithm in terms of fast convergence and accuracy of the ANN model.

5. Proposed method

In this paper, a new metaheuristic algorithm called TLBO is used to obtain better FLANN learning model. The problem solving strategy of TLBO along with Gradient Descent Learning is used to find the best set of weights for FLANN model for classification task. The objective is to investigate performance of TLBO to enhance learning capability of FLANN classification model as compared to GA, PSO and HS. The

pseudo codes developed during implementation of TLBO based FLANN (TLBO-FLANN) is presented in this section. In this paper, the simulation results and the comparisons of performance of these hybrid FLANN models (GA-FLANN, PSO-FLANN, HS-FLANN and the proposed TLBO based FLANN (TLBO-FLANN)) are represented and discussed. Table 2 represents various notations used for description of the proposed schemes and designing algorithms.

5.1. TLBO based gradient descent learning FLANN

Initially, the population of weight-sets X (population of students) is initialized with 'n' no.s of weight-sets for FLANN. Each weight-set in the population X is a vector of weights

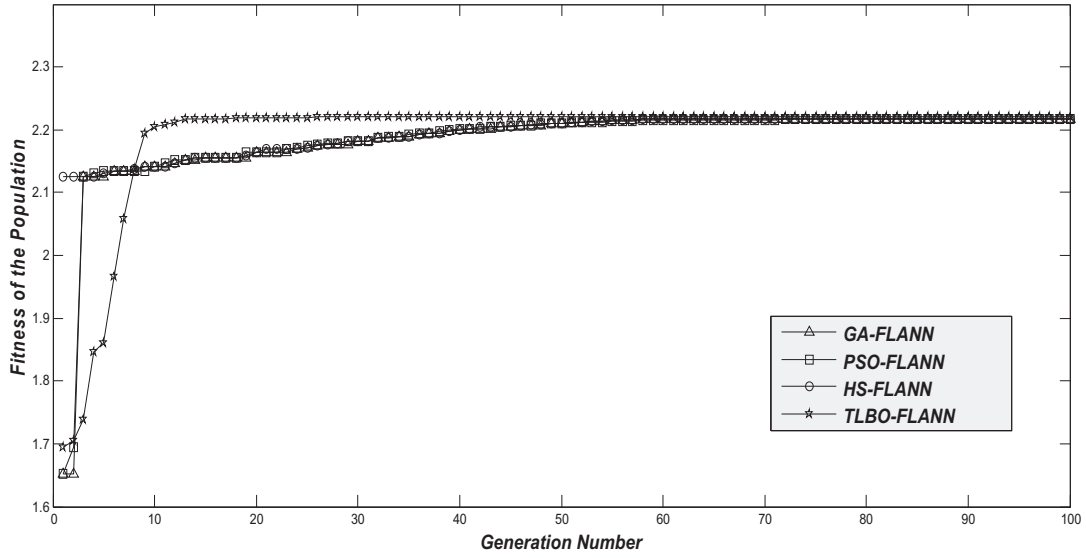


Figure 9 Observation on improvements in fitness of population in various iterations on Pima dataset.

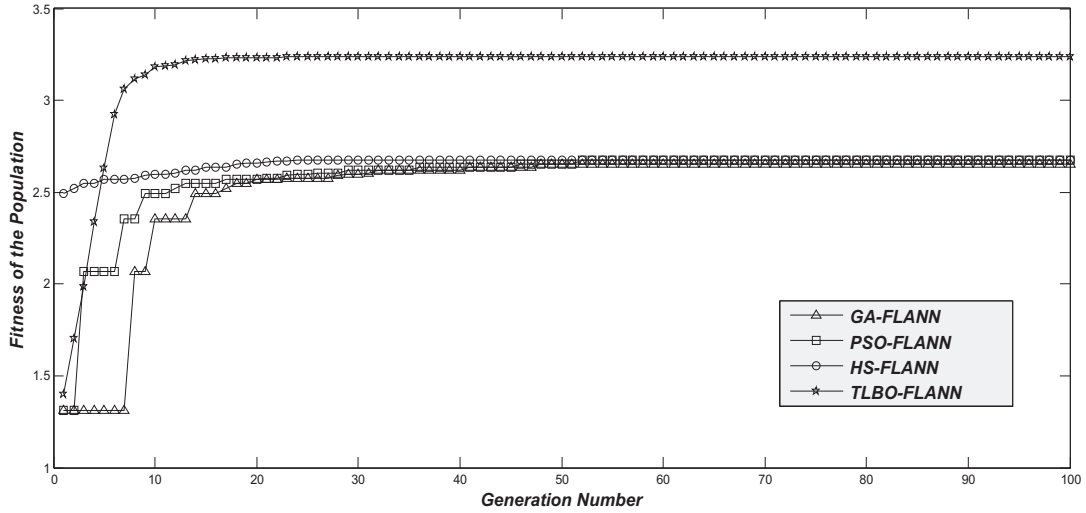


Figure 10 Observation on improvements in fitness of population in various iterations on New Thyroid dataset.

initialized randomly between  $-1$  and  $1$  which are the potential candidate weight-sets of FLANN model of a particular data set. Each individual weight-set in  $\mathbf{X}$  can be defined as:

$$x_i = (w_{i,1}, w_{i,2} \dots w_{i,m \times a \times (2 \times k + 1)}) \quad (1)$$

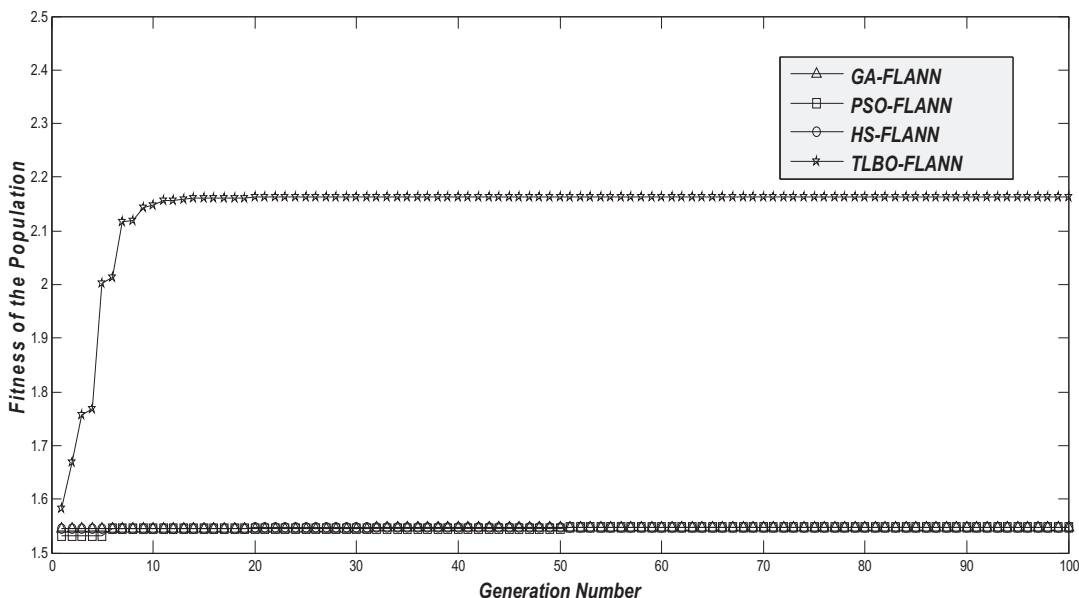
$$\mathbf{X} = (x_1, x_2 \dots x_n) \quad (2)$$

In Eq. (1), the  $(2 \times k + 1)$  is the No. of functionally expanded values for a single value in input pattern (for a chosen value of  $k$ ), ‘ $a$ ’ is the number of values (attributes) in a single input pattern, ‘ $m$ ’ is the number of patterns in the data set and ‘ $n$ ’ is the number of weight-sets in the population  $\mathbf{X}$ . The set of weight-sets in  $\mathbf{X}$  are represented as in Eq. (2). Here, the objective is to prune out optimal weight-sets for the FLANN model to obtain better classification accuracy. Each weight-set  $x_i$  is set to FLANN individually and the FLANN

model is trained with a particular data set. Based on the output of FLANN and given target value, the error of the model is obtained. For a specific data set, the root mean square error (RMSE) (Eq. (3)) for each weight-set  $x_i$  is computed using output of the FLANN and given target value (Algorithm 2). Based on RMSE’s, fitness of the weight-sets are computed by using Eq. (4). The RMSE of predicted output values  $\hat{y}_i$  of a target variable  $y_i$  is computed for ‘ $n$ ’ different predictions as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}} \quad (3)$$

$$F_{x_i} = \frac{1}{RMSE_i} \quad (4)$$

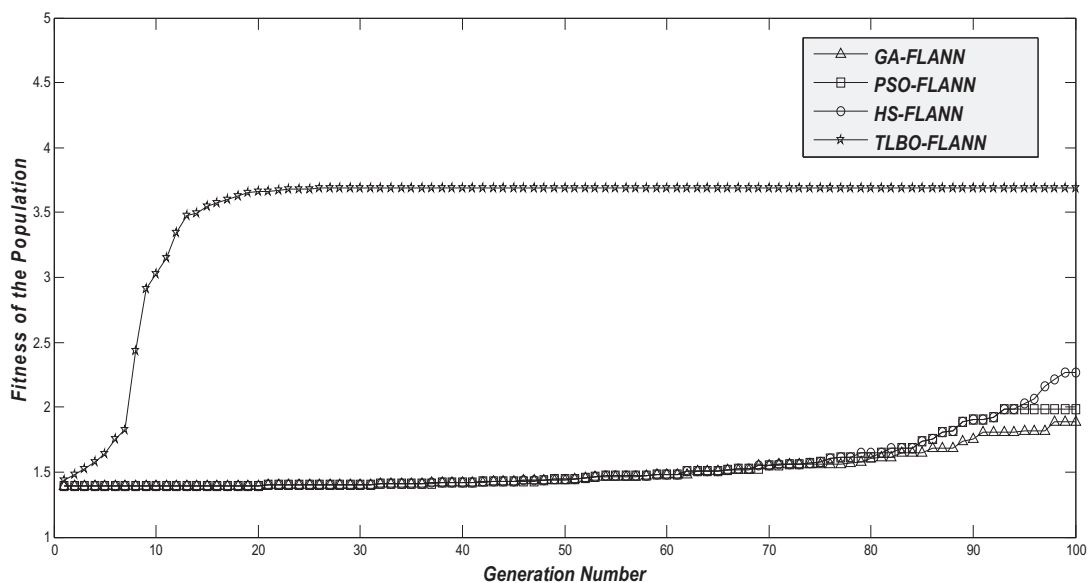


**Figure 11** Observation on improvements in fitness of population in various iterations on Bupa dataset.

In Eq. (4),  $x_i$  is the  $i$ th weight-set in the population,  $RMSE_i$  is the root mean square error of  $i$ th weight-set and  $F_{x_i}$  is the fitness of  $i$ th weight-set  $x_i$ .

After evaluation of fitness values for each weight-set in  $X$ , the weight-set with maximum fitness is selected as Teacher ( $x_{teacher}$ ). From the population of  $X$ , the mean of the weight-sets ( $X_{mean}$ ) is computed by calculating the mean of all the weight-sets in  $X$ . After the calculation of teaching factor ( $T_F$ ) (Algorithm-1), the next population  $X_{next}$  is generated from  $X$ ,  $X_{mean}$ ,  $x_{teacher}$  and  $T_F$  (Step-4 of Algorithm-1). Then the weight-sets in initial population  $X$  are updated by comparing

fitness of weight-sets in  $X$  and  $X_{next}$  (Step-5 of Algorithm-1). The resultant population of weight-sets  $X$  goes through improvisation steps (Step-6 of Algorithm-1) in which two weight-sets are randomly selected from the population  $X$  and best among them are chosen as weight-set for the next generation  $X_{next}$  by comparing their fitness, thereby giving more chances to migrate better weight-sets for the next generation. These processes are continued until maximum iteration is reached or increase in fitness of weight-sets in  $X$  is not significant. The complete flow of execution can be realized in Fig. 2.



**Figure 12** Observation on improvements in fitness of population in various iterations on Dermatology dataset.

**Algorithm 1.** TLBO based gradient descent learning FLANN (TLBO-FLANN)

---

```

1. Initialize n number of weight-sets (Population of students).
 $X = \{x_1, x_2, \dots, x_n\}$ , where each  $x_i$  is a potential weight-set of
FLANN model which is initialized randomly as
 $x_i = \{w_{i,1}, w_{i,2}, \dots, w_{i,m \times a \times (2 \times k + 1)}\}$ .
2. Calculate mean of all the weight-sets ( $x_{mean}$ ) in the population
X.
3. Compute fitness of all the weight-sets in X by using Algorithm-
2 (fitfromtrain procedure) and select weight-set with maximum
fitness as best weight-set ( $x_{teacher}$ ).
4. Generate next population  $X_{next}$  by using weight-sets in old
population X,  $x_{mean}$ ,  $x_{teacher}$  and  $T_F$ :
  for  $i = 1:1$ : nos of weight-sets in the population X
     $T_F = round(1 + rand(1))$ 
     $x_i = x_i + rand(1)(x_{teacher} - T_F * x_{mean})$ 
     $x_{next}(i) = x_i$ 
  endfor
5. Update population of weight-sets X by comparing fitness of
weight-sets in X and  $X_{next}$ :
  for  $i = 1:1$ : nos of weight-sets in the population X
    if  $(X(i) < X_{next}(i))$ 
       $X(i) = X_{next}(i)$ 
    endif
  endfor
6. Randomly select two weight-sets from the population and
improve them:
  for  $k = 1:1$ : nos of weight-sets in the population X
    Select  $i$ th and  $j$ th weight-sets  $x_i$  and  $x_j$  randomly from
population X.
    Compute fitness of  $x_i$  by using algorithm-2 as
 $F_i = \text{fitfromtrain}(\varphi, x_i, t, \mu)$ .
    Compute fitness of  $x_j$  by using algorithm-2 as
 $F_j = \text{fitfromtrain}(\varphi, x_j, t, \mu)$ .
    If  $(F_i < F_j)$ 
       $x_{new}(i) = x_i + rand(1)(x_j - x_i)$ 
    Else
       $x_{new}(j) = x_j + rand(1)(x_i - x_j)$ 
    Ifend
  Endfor
7. Check for termination criteria:
  if (maximum nos of generation reached)
    then go to step-8.
  else go to step-2
  endIf
8. Exit

```

---

**Algorithm 2.** fitfromtrain procedure**Function F** = fitfromtrain ( $\varphi, x_i, t, \mu$ )

Let ' $\varphi$ ' be the functionally expanded input data and ' $x_i$ ' be the selected weight-set from the population. The vector  $S$  is computed as  $S = \varphi \cdot x_i$ .

The output vector ' $y$ ' is computed by using tanh activation function as  $y = \tanh(S)$ .

The errors of the network ' $e$ ' is computed by using target vector ' $t$ ' and output vector ' $y$ ' as  $e = t - y$

Compute error term  $\delta(k) = \left(\frac{1-y_k^2}{2}\right) \times e(k)$ , for  $k = 1, 2, \dots, L$

where  $L$  is the number of patterns

If  $\varphi = (\varphi_1, \varphi_2 \dots \varphi_L)$ ,  $e = (e_1, e_2 \dots e_L)$  and  $\delta = (\delta_1, \delta_2 \dots \delta_L)$  are vectors which represent set of functional expansion, set of errors and set of error terms respectively, then weight factor of  $w$  ' $\Delta W$ ' is

$$\text{computed as follows: } \Delta W_q = \left( \frac{\sum_{i=1}^L 2 \times \mu \times \varphi_i \times \delta_i}{L} \right)$$

Compute root mean square error (RMSE) by using Eq. (3) from target vector ' $t$ ' and output vector ' $y$ '

Compute fitness ' $F$ ' of the network instance of FLANN model as  $F = 1/\text{RMSE}$  (Eq. (4)).

**end**

---

**6. Experimental setup & data preparation**

In this section, the environment for simulation, the data set used for training & testing phase and parameter settings for proposed methods during simulation are presented. All the hybrid models (GA-FLANN, PSO-FLANN, HS-FLANN and TLBO-FLANN) are implemented in MATLAB (Version 9.0) in a system with Window XP operating system. After obtaining the results of simulation, statistical analysis has been carried out using SPSS statistical tool (Version 16.0). The benchmark data sets (Table 3) used for training and testing phase for classification are originated from UCI machine learning repository (Bache et al., 2013) and processed by KEEL software (Alcalá-Fdez et al., 2011). The detailed descriptions about all these data sets can be obtained at '<http://archive.ics.uci.edu/ml/>' and '<http://keel.es/>'.

**6.1. Cross validation**

In this paper, all the data sets used for classification are prepared for cross validation by using 5-fold cross validation technique (Larson, 1931; Mosteller et al., 1968). During the preparation of data sets for the 5-fold cross validation, 5 pairs of data set samples are created and each pair contains data sets for the training and testing phase. For example (Table 4), the 'newthyroid-5-1tra.dat' and 'newthyroid-5-1ts t.dat' data are a pair of data sets sample of New Thyroid data set which are used for the training and testing phase for a single run respectively. As 5-fold cross validation is employed, the New Thyroid data sets contain 5 such pairs of data set samples for training and testing the algorithms. The 5-fold cross validated data set for New Thyroid data set is presented in Table 4. All other data sets are prepared for 5-fold cross validation in the same way and collected from KEEL data set repository. The maximum fitness from their respective RMSEs on 5-fold cross validation data set during training and testing phase is listed in Table 5. The maximum fitness values obtained by various algorithms on training data ('newthyroid-5-1tra.dat', 'newthyroid-5-2tra.dat', 'newthyroid-5-3tra.dat', 'newthyroid-5-4tra.dat' and 'newthyroid-5-5tra.dat') for training phase and testing data ('newthyroid-5-1tst.dat', 'newthyroid-5-2tst.dat', 'newthyroid-5-3tst.dat', 'newthyroid-5-4tst.dat' and 'newthyroid-5-5tst.dat') for testing phase and are posted. This process is repeated for all other data sets in Table 3 and maximum fitness is posted in Table 5.

6.2. Parameter setting used for simulation

The FLANN parameters and TLBO parameters are set to suggested values from previous related works (Rao et al., 2011; Dehuri et al., 2012; Naik et al., 2014; Pao et al., 1989) on trial and error basis.

**FLANN parameter:**  
 For FLANN, the learning parameter 'μ' is set to 0.13 in gradient descent learning by testing the models in the range 0–3. For functional expansion in FLANN, the value of n is set to 5, thereby each value in the input pattern is expanded to 11 number of functionally expanded input values (As in FLANN model, it is suggested (Pao et al., 1989; Pao and Takefuji, 1992) to generate 2n + 1 number of functionally expanded input values for a single value in the input pattern). The number of functionally input values increases hugely if a larger value of n is selected and the small value of n is unable to handle the non-linear nature of real world data sets. To make a balance between them, we have used the tested and suggested value of n (Dehuri et al., 2012; Naik et al., 2014) for functional expansion

**TLBO parameter:**  
 For TLBO, we have set the common algorithmic parameters by testing the model by considering the suggested values (Population size = 40; Number of generations = 100; Stopping Criteria = Maximum number of generation) from previous related works (Rao et al., 2011)

7. Result obtained

Table 5 describes the comparison of maximum fitness obtained by all models and Table 6 represents average RMSEs obtained by four models: GA-FLANN, PSO-FLANN, HS-FLANN and TLBO-FLANN on all 11 no of data sets. In this study, the performance of GA, PSO, HS and TLBO are examined in order to know the improvement of weight-sets in the population by these algorithms in various iterations. The changes in fitness of weight-sets in different iterations are observed in all the 11 number of data sets and the Figs. 3–13 demonstrates the improvements of fitness of weight-sets in the population.

8. Proof of statistical significance of the results

In this section, the statistical comparison of all the models over multiple data sets (Demersar, 2006) is presented to argue that the projected method is statistically better and significantly different from other alternative methods by using Friedman test (Friedman, 1937, 1940). List of results on which these tests have been carried out is presented in Table 7.

8.1. Friedman test

The Friedman test is a non-parametric statistical method which computes average ranks of algorithms by using Eq. (5) and compares them.

$$R_j = \frac{1}{N} \sum_i r_i^j \tag{5}$$

In Eq. (5),  $r_i^j$  is the rank of the  $j$ th of  $k$  number of models on  $i$ th of  $N$  number of data sets. In Table 7, the ranks of each model on various data sets are shown in braces. Based on  $r_i^j$ , the average ranks of four models are found out from equation 5. The average ranks for all models: GA-FLANN,

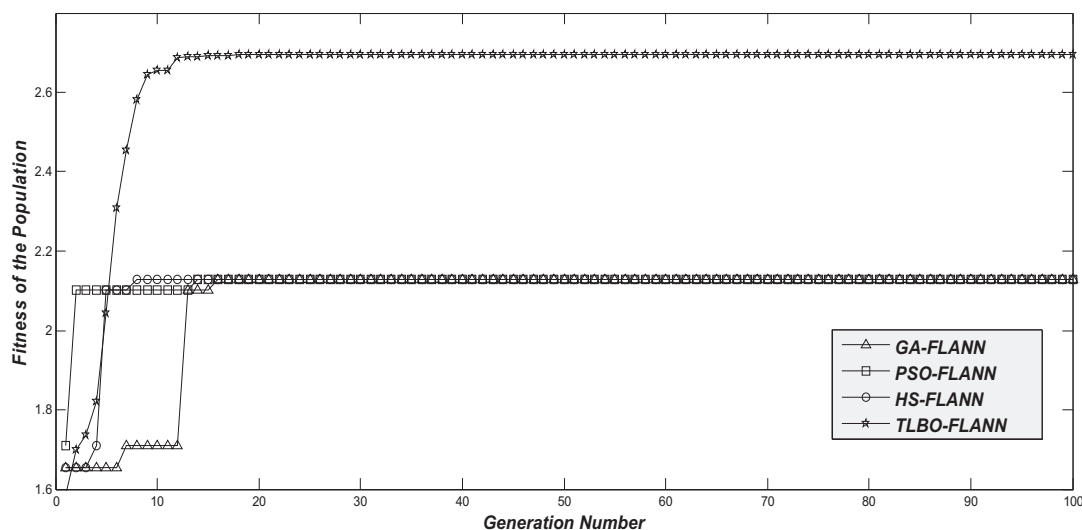


Figure 13 Observation on improvements in fitness of population in various iterations on Heart dataset.

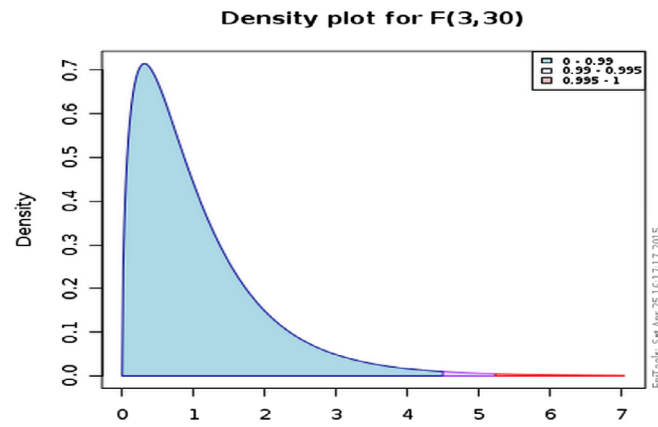


Figure 14 Density plot for 4.514  $F_F$  statistic and (3, 30) degree of freedom by selecting  $\alpha = 0.01$ .

Table 8 Result of Holm statistical test.

$i$	Learning models	$z$ -values	$p$ -values	$\frac{\alpha}{(k-i)}$
1	TLBO-FLANN : GA-FLANN	3.969	0.000036	0.0033
2	TLBO-FLANN : PSO-FLANN	3.141	0.000842	0.005
3	TLBO-FLANN : HS-FLANN	1.32	0.093418	0.01

PSO-FLANN HS-FLANN and TLBO-FLANN are found as  $\{R_1 = 3.455, R_2 = 3, R_3 = 2, R_4 = 1.272\}$  respectively. The  $X_F^2$  value is computed from average rank  $R_j$  of each model by using Eq. (6). In this study, we got the value of  $X_F^2$  as 10.263. From the value of  $X_F^2$ , the Friedman statistics  $F_F$  is computed by Eq. (7) and found as 4.514. The Friedman statistics are distributed according to  $X_F^2$  with  $(k - 1)$  degree

of freedom under the null-hypothesis ( $H_0$ ) and the critical value of the F-distribution can be obtained from  $F_F$  with  $(k - 1)$  and  $(k - 1) * (N - 1)$  degree of freedom. In our case, for the four number of classifiers and 11 number of data sets, the  $F_F = 4.514$  with  $4 - 1 = 3$  and  $(4 - 1) * (11 - 1) = 30$  degrees of freedom and the crucial value = 4.510 is obtained from suitably selecting  $\alpha = 0.01$ . Density plot for degree of freedom (3, 30) is obtained and displayed in Fig. 14. The null-hypothesis is clearly rejected, as the critical value 4.510 is less than  $F_F$  statistic (4.514).

$H_0$ : All the learning models have the same rank and differences between them are merely random, hence they are equivalent.

$$X_F^2 = \left( \frac{12N}{k(k+1)} \right) \left( \sum_j R_j^2 - \frac{k(k+1)^2}{4} \right) \tag{6}$$

Descriptives

Sample	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
GA-FLANN	110	2.4494	1.17060	.11161	2.2282	2.6706	1.55	5.97
PSO-FLANN	110	2.4689	1.16482	.11106	2.2488	2.6891	1.55	5.97
HS-FLANN	110	2.5278	1.16891	.11145	2.3069	2.7487	1.55	5.97
TLBO-FLANN	110	3.7371	2.45325	.23391	3.2735	4.2007	2.06	11.27
Total	440	2.7958	1.67557	.07988	2.6388	2.9528	1.55	11.27

ANOVA

Sample	Sum of Squares	df	Mean Square	F	Sig.
Between Groups (Combined)	130.310	3	43.437	17.182	.000
Linear Term	84.595	1	84.595	33.464	.000
Deviation	45.714	2	22.857	9.042	.000
Within Groups	1102.195	436	2.528		
Total	1232.505	439			

Figure 15 Results of a one-way-ANOVA statistical test.

**Multiple Comparisons**

Dependent Variable: Sample

	(I) Algorithm	(J) Algorithm	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	GA-FLANN	PSO-FLANN	-.01951	.21439	1.000	-.5724	.5334
		HS-FLANN	-.07840	.21439	.983	-.6313	.4745
		TLBO-FLANN	-1.28765*	.21439	.000	-1.8406	-.7348
	PSO-FLANN	GA-FLANN	.01951	.21439	1.000	-.5334	.5724
		HS-FLANN	-.05889	.21439	.993	-.6118	.4940
		TLBO-FLANN	-1.26814*	.21439	.000	-1.8210	-.7152
	HS-FLANN	GA-FLANN	.07840	.21439	.983	-.4745	.6313
		PSO-FLANN	.05889	.21439	.993	-.4940	.6118
		TLBO-FLANN	-1.20925*	.21439	.000	-1.7622	-.6563
	TLBO-FLANN	GA-FLANN	1.28765*	.21439	.000	.7348	1.8406
		PSO-FLANN	1.26814*	.21439	.000	.7152	1.8210
		HS-FLANN	1.20925*	.21439	.000	.6563	1.7622
Dunnnett t (2-sided) <sup>a</sup>	GA-FLANN	TLBO-FLANN	-1.28765*	.21439	.000	-1.7930	-.7823
	PSO-FLANN	TLBO-FLANN	-1.26814*	.21439	.000	-1.7735	-.7628
	HS-FLANN	TLBO-FLANN	-1.20925*	.21439	.000	-1.7146	-.7039

\*. The mean difference is significant at the 0.05 level.

a. Dunnnett t-tests treat one group as a control, and compare all other groups against it.

**Figure 16** Results of Tukey and Dunnnett statistical test.

**Sample**

	Algorithm	N	Subset for alpha = 0.05	
			1	2
Tukey HSD <sup>a</sup>	GA-FLANN	110	2.4494	
	PSO-FLANN	110	2.4689	
	HS-FLANN	110	2.5278	
	TLBO-FLANN	110		3.7371
	Sig.		.983	1.000
Duncan <sup>a</sup>	GA-FLANN	110	2.4494	
	PSO-FLANN	110	2.4689	
	HS-FLANN	110	2.5278	
	TLBO-FLANN	110		3.7371
	Sig.		.733	1.000

Means for groups in homogeneous subsets are displayed.

a. Uses Harmonic Mean Sample Size = 110.000.

**Figure 17** Observations on models in their homogeneous group based on their level of significance.

**Table 9** Various notations used in time complexity analysis on various models.

Symbol used	Meaning
T(n)	Time complexity of the models on input of n number of solution vector (weight-set)
O	Big-oh asymptotic notation
n	Number of weight-sets in the population.
nc	Number of attributes in weight-sets
c	Constant time
j, l	Number of functionally extended input patterns
k	Number of features in each functionally extended input pattern

$$F_F = \frac{((N-1)X_F^2)}{(N(K-1) - X_F^2)} \quad (7)$$

After the rejection of null-hypothesis from Friedman test, the post hoc test has been carried out by using the Holm procedure (Demsar, 2006; Luengo et al., 2009; Garcia et al., 2010) in order to compare the performance of the proposed model with other models based on z-score value and p-value.

### 8.2. Holm procedure

In this section, the Holm procedure (Holm, 1979) is used for pair wise comparison based on their z-score value and p-value. The Eq. (8) is used to obtain z-value and p-value is computed from the z-value and table of the normal distribution.

$$z = \frac{(R_i - R_j)}{\sqrt{\frac{k(k+1)}{6N}}} \quad (8)$$

Here, z is the z-score value, k is the number of models and N is the number of data sets. The  $R_i$  and  $R_j$  are average ranks of  $i$ th and  $j$ th model respectively. All four models are compared based on z-value, p-value and  $\frac{\alpha}{(k-i)}$  (Table 8), where 'i' is the model's number. By using the Holm test, while comparing  $p_i$  values with corresponding  $\frac{\alpha}{(k-i)}$  values, we observed that, in almost all cases  $p_i$  is less than  $\frac{\alpha}{(k-i)}$  (except comparison between TLBO-FLANN with HS-FLANN). Hence, on this basis the null-hypothesis is rejected. Thus, the proposed classification model TLBO-FLANN is statistically better and significantly different from other models (except HS-FLANN). While comparing with HS-FLANN, the

TLBO-FLANN is found to be better but not significantly better than HS-FLANN because, p-value is not less than respective  $\frac{\alpha}{(k-i)}$  value.

### 8.3. Post-hoc ANOVA statistical analysis (Tukey test & Dunnett test)

After the rejection of the null-hypothesis from the Friedman test in the Section 8.1 and analysis under the Holm procedure in Section 8.2, in this section, the ANOVA statistical analysis (Fisher, 1959) has been carried out by using Tukey Test & Dunnett test to get generalized statistics on the performance of all models.

In this paper, the statistics on all model's performance is computed under ANOVA test by using SPSS (Version: 16.0) statistical tool. All the methods are executed for ten no.s of runs on each data set. The test has been carried out with 95% confidence interval, 0.05 significant level and linear polynomial contrast (Fig. 15). To get the differences between the performances of classifiers, the Post-hoc test is conducted by using Tukey test (Tukey, 1949) and Dunnett test (Dunnett, 1980). The Tukey test is carried out for comparisons of performance of all models with each other and the Dunnett test for comparison of all models with base classification model (proposed model). The results from Tukey test and Dunnett test are presented in Fig. 16. As a conclusion of these tests, we noticed that, the mean differences (between-classification models variability) among models are larger than the standard errors (between-error variability) (Fig. 16). Also in Dunnett test (Fig. 16), we observed the same as that of Tukey test. Hence the null-hypothesis can be clearly rejected. Furthermore, all the models in their homogeneous group based on their level of significance are presented in Fig. 17.

## 9. Time complexity analysis

In this section, all the models that we have considered are analyzed by comparing their time complexities. The notations used for time complexity analysis are listed in Table 9. Tables 10–13 represent the number of program steps taken by the models: GA-FLANN, PSO-FLANN, HS-FLANN and TLBO-FLANN respectively. In this analysis, we noted that time complexity T(n) of GA-FLANN, PSO-FLANN, HS-FLANN and TLBO-FLANN are dominated by the factor  $(n * (c * ((j * k * c) + (l * c))))$ ,  $(c * (n * (c * ((j * k * c) + (l * c))))$ ,  $(c * (n * (c * ((j * k * c) + (l * c))))$  and  $(c * (n * (c * ((j * k * c) + (l * c))))$  respectively. The time complexities

**Table 10** Time complexity analysis on GA based FLANN (GA-FLANN) model.

Algorithm step	Time complexity
Fitness evaluation in FLANN model	$(n * (c * ((j * k * c) + (l * c))))$
Mating pool construction	$(n + (n + c))$
Crossover	$((n/2) * c)$
Selection	$(n * c)$
Stopping criteria check	$(n * c)$
Overall time complexity	$T(n) = (n * (c * ((j * k * c) + (l * c)))) + (n + (n + c)) + ((n/2) * c) + (n * c) + (n * c)$ $T(n) = O((n * (c * ((j * k * c) + (l * c))))$



**Table 11** Time complexity analysis on PSO based FLANN (*PSO-FLANN*) model.

Algorithm step	Time complexity
Fitness evaluation in FLANN model	$(n * (c * ((j * k * c) + (l * c))))$
Local best selection	$(2 * (n * (c * ((j * k * c) + (l * c)))) + (n * c))$
Global best selection	$(n * c)$
Velocity calculation	$(n * n * c * c)$
Particle position updation	$(n * c)$
Stopping criteria check	$(n * c)$
Overall time complexity	$T(n) = (n * (c * ((j * k * c) + (l * c)))) + (2 * (n * (c * ((j * k * c) + (l * c)))) + (n * c)) + (n * c) + (n * n * c * c) + (n * c) + (n * c)$ $T(n) = O(c * (n * (c * ((j * k * c) + (l * c))))$

**Table 12** Time complexity analysis on HS based FLANN (*HS-FLANN*) model.

Algorithm step	Time complexity
Initialization of harmony memory (HM), harmony memory size, harmony memory consideration rate, pitch adjustment rate and, bandwidth	$c$
Computes MCP (memory consideration probability), PAP (pitch adjustment probability) and RP (randomization probability).	$c$
Improvise HM to get new harmony memory (NHM)	$(HMS * c)$
Update the HM based on comparison between solution vectors of HM and NHM	$(HMS * c * (n * (c * ((j * k * c) + (l * c))))$
Fitness evaluation in FLANN model	$(n * (c * ((j * k * c) + (l * c))))$
Stopping criteria	1
Overall time complexity	$T(n) = c + c + (HMS * c) + (HMS * c * (n * (c * ((j * k * c) + (l * c)))) + (n * (c * ((j * k * c) + (l * c)))) + 1$ $T(n) = O(c * (n * (c * ((j * k * c) + (l * c))))$

**Table 13** Time complexity analysis on TLBO based FLANN (*TLBO-FLANN*) model.

Algorithm step	Time complexity
Mean calculation	$(n * c)$
Fitness evaluation in FLANN model	$(n * (c * ((j * k * c) + (l * c))))$
Teacher selection	$(n * c)$
Next population generation	$(n * c)$
Population updation	$(n * c)$
Improvisation of population	$((n * 2 * (c * ((j * k * c) + (l * c)))) + (n * c))$
Stopping criteria	1
Overall time complexity	$T(n) = (n * c) + (n * (c * ((j * k * c) + (l * c)))) + (n * c) + (n * c) + (n * c) + ((n * 2 * (c * ((j * k * c) + (l * c)))) + (n * c)) + 1$ $T(n) = O(c * (n * (c * ((j * k * c) + (l * c))))$

**Table 14** Comparison of various models based on time complexity.

Implemented algorithms	Time complexity
GA-FLANN	$O((n * (c * ((j * k * c) + (l * c))))$
PSO-FLANN	$O(c * (n * (c * ((j * k * c) + (l * c))))$
HS-FLANN	$O(c * (n * (c * ((j * k * c) + (l * c))))$
TLBO-FLANN	$O(c * (n * (c * ((j * k * c) + (l * c))))$

of all the algorithms are listed in Table 14. Here, we conclude that the time complexity of all the models is dominated by the time complexity of the basic step 'Fitness Evaluation in FLANN model' (Algorithm-2) which is  $(n * (c * ((j * k * c) + (l * c))))$ . The proposed method TLBO-FLANN is equivalent

with other models (GA-FLANN, PSO-FLANN and HS-FLANN) in terms of time complexity but it is better in learning ability as compared to other alternatives GA-FLANN, PSO-FLANN and HS-FLANN.

## 10. Discussion & conclusion

The proposed TLBO-FLANN model can be computed at a low cost due to that less complex architecture of FLANN and TLBO requires few common algorithmic specific parameters and mathematical computations. Also TLBO is free from complicated operators (like crossover in GA) and parameters (like c1, c2 in PSO and HMCR, PAR & bw in HS). The performance of TLBO-FLANN can be improved by adopting other improved versions of TLBO.

In this study, we analyzed the proposed TLBO-FLANN by using various statistical methods like Friedman test, Holm procedure, Tukey & Dunnett Test post hoc test and one-way-ANOVA. As a result, a clear rejection of null-hypothesis is noticed in Friedman Test and Tukey & Dunnett test post hoc test, but in the Holm procedure, the null-hypothesis is rejected except between TLBO-FLANN and HS-FLANN. This is because; the proposed TLBO-FLANN model is statistically better and significantly different from GA-FLANN and PSO-FLANN models. On the other hand, the TLBO-FLANN outperforms HS-FLANN in terms of accuracies but not significantly different and higher than HS-FLANN. From rigorous tests under well known statistical methods (Friedman test, Post-hoc test by Holm procedure, Tukey test, Dunnett test and one-way-ANOVA), we claim the proposed TLBO-FLANN model is better and outperforms other alternative hybrid models (GA-FLANN, PSO-FLANN and HS-FLANN).

In time complexity analysis, it is observed that, the proposed method TLBO-FLANN is similar to other alternatives (GA-FLANN, PSO-FLANN and HS-FLANN), but the performance of TLBO-FLANN is better than GA-FLANN, PSO-FLANN and HS-FLANN in term of learning capability. The future work is comprised of integration of other new and improved variants of TLBO with FLANN in diverse applications of data mining.

#### Acknowledgement

We would like to thank anonymous reviewers for their valuable comments and suggestions which have certainly improved the quality of this paper. This work is supported by Department of Science & Technology (DST), Ministry of Science & Technology, New Delhi, Govt. of India, under Grants No. DST/INSPIRE Fellowship/2013/585.

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