

## **Electronic Circuit Simulation of Ellias-grossberg Oscillators**

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**Abstract.** This paper deals with modeling and simulation of neural oscillators introduced by Ellias and Grossberg. The proposed model incorporates all-important physiological characteristics. The model uses reliable, compact, and inexpensive components for simulation and several such models can be interconnected together to study complex dynamic behavior of central nervous system such as the human gait transitions. The model has been successfully tested for different phenomena and behavior of neuronal systems. A chain of the proposed electronic model tested for phase locking produced good results similar to those obtained from software simulation. This model is also a step in the direction of development of artificial intelligent systems.

### **1. Introduction**

An electronic model of any system possesses advantages of being compact, flexible, and cheap. An electronic model of a neuron gives clear and consistent picture of its physiological behavior. In such a model various circuit parameters, which are analogous to different physiological processes, can be varied and the effect of these changes on the performance of the system can be visually observed. Such studies are of great importance in research, diagnosis and prosthesis. This paper presents an electronic version of a model proposed by Ellias and Grossberg [1] (EG oscillator) which provides an accurate functional representation of reciprocally connected fast-slow oscillator. Several such models can be connected together to study the behavior of complex interconnected neuronal networks.

The model incorporates all-important physiological characteristics. The model uses reliable, compact, and inexpensive components. The model has been successfully tested for different phenomena and behavior of neuronal systems. A chain of the

purposed electronic model tested for phase locking produced good results similar to those obtained from software simulation. This model is also a step in the direction of development of artificial intelligent systems.

**The model incorporates the following features:**

- 1- The use of integrated circuits facilitates the construction of a reliable, compact and reasonably inexpensive units of the model.
- 2- Operational amplifiers are extensively used so that the signals at each stage bear a simple mathematical relationship to those at earlier stages and thus permitting easy comparison with actual neuronal data.
- 3- All-important properties of a neuronal element can be simulated.

In this paper we provide simulation results for the electronic version of the Ellias-Grossberg single-oscillator using Electronics Workbench-5. We also study the behavior of a chain of four-coupled neural oscillators [2, 3] and provide EWB simulation results. We then study the effect of noise [4], which shows that the behavior of the oscillator is stable in the presence of noise. We also propose a circuit for human walk-and-run gait transition [5] and demonstrate its behavior through electronic simulation.

## 2. Single EG Oscillator

Neurons are the information generation, transmission and distribution elements in the central nervous system of the living creatures. They generate, store, update and process information in the brain. Motor neurons carry information from brain and spinal cord to different parts of the body and the sensory neurons carry information from different parts of the body to spinal cord of the brain. The size and shape of a neuron may vary from one location to another due to variation in function. A number of models have been developed to simulate the characteristics of a neuron. The complexity of the models increase with the increase in the number of variables in order to describe accurately the time course of the ionic current that follow across nerve membranes [6, 7]. In this work, Ellias-Grossberg model [1] has been chosen because of its simplicity compared to other biological models, ease of control due to its parameters, and insensitivity to noise. The model of single oscillator is defined in the simplest form as a feedback loop between an excitatory unit and an inhibitory unit as shown in Fig. 1.a and governed by the following equations:

$$\frac{dx}{dt} = -Ax + (B - x)\{C[x - \Gamma]^+ + I\} - Dx[y - \Gamma]^+ \quad (1)$$

$$\frac{dy}{dt} = E(x - y) \quad (2)$$

where  $(s)^+ = \max(s, 0)$ . The variable  $x$  represents the potential of an excitatory cell governed by a non-linear shunting equation.  $y$  represents the potential of an inhibitory cell governed by a linear equation. The parameters are  $A=1$ ,  $B=1$ ,  $C=20$ ,  $D=33.3$ ,  $\Gamma=0.5$  and  $I$  ranged from 0.2 to 1.0 in different trials.  $E$  in (2) governs the relative time scales of  $x$  and  $y$  and represents the relative rate at which the inhibitory interneuron tracks the firing rate of the excitatory cell. When  $E$  is small and thus (2) is in its relaxation regime, the excitatory cell exhibits a spike like waveform and when  $E$  is near unity the excitatory cell exhibits sinusoidal behavior. Computer simulations were performed for the model, and the obtained output response is shown in Fig. 1.b, c.

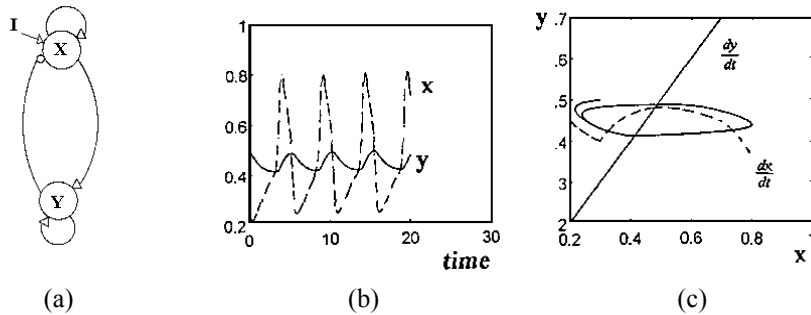
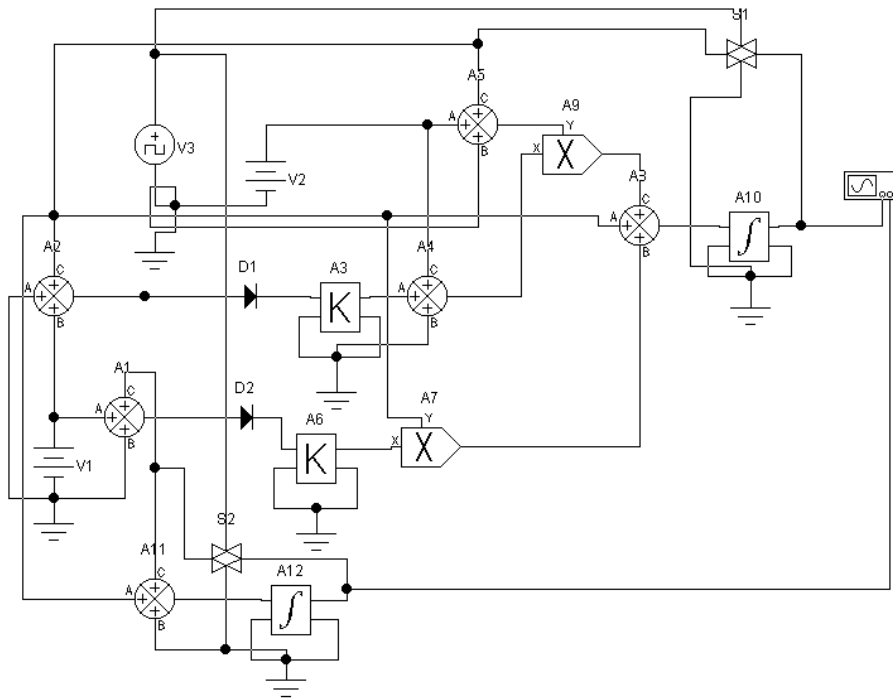


Fig.1. (a) EG Oscillator shows the connections between the excitatory and inhibitory units. Triangle represents excitatory connection and circle represent inhibitory connection. (b) and (c) are the simulation results of the Ellias-Grossberg model, where (b) Waveforms for inhibitory cell (solid curve) and excitatory cell (dashed curve), (c) Phase portrait, limit cycle and Nullclines.

### 3. Electronic Simulation of EG Oscillator

Electronics Workbench-5 was used to simulate the single EG oscillator. Fig. 2 is the schematic diagram for the EG Oscillator. As shown in the circuit, the input is driven through a battery V2 that introduces an active input  $I=1$ . The input battery value controls the activity of the neuron. For normal operation the input battery is replaced by an external input  $I$ . The input summer A5 collects the coming inputs from other neurons and the feedback signal from the neuron output. The battery V1 presents the threshold value used ' $\Gamma$ '. The diodes D1 and D2 used in the circuit resemble the solution for the  $S^+$  problem, so its output is the maximum of 0 or  $S$  that means the output has to be +ve in all cases. The forward voltage drop for the used diode has taken into consideration in simulation trials. The gain blocks represent the parameter values  $C$ , and  $D$ . The parameter  $E$  value is used as the gain of the integrator block A12. The feedback path is fed to the input summer through the analog switches, which are controlled by the pulse generator V3. The analog switch is used to feed the output at certain times to the input summer and controls the base frequency of the circuit. The output from the integrator

A10 resembles the excitatory and the output from the integrator A12 resembles the inhibitory.



**Fig.2. Schematic diagram for a single EG oscillator.**

From our trials on simulating this model, it was shown that changing the Operational Amplifier model affects the output very much. We were not concerned about component speed because of the low speed nature of our signal and the Ultra high-speed components available now from everywhere [8 - 11].

The simulation results are as shown in Fig. 3, for the excitatory and inhibitory cells activities, and the phase plane portrait that shows the limit cycle starting from a random value depending on the initial conditions of the excitatory and the inhibitory outputs. Changing the initial conditions in the range from 0 to 1 does not affect the limit cycle at all. These boundaries are chosen according to the simulated neuron output limits.

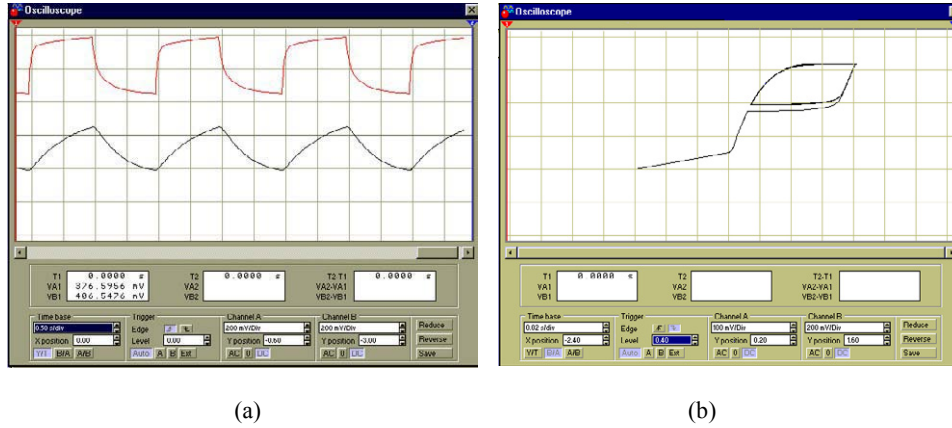


Fig.3. Relaxation oscillation obtained for the simulated Hardware neuron. (a) Waveforms for inhibitory cell (lower curve) and excitatory cell (upper curve), (b) Phase portrait, limit cycle.

#### 4. A chain of Four EG Oscillators

A chain of four coupled oscillators was simulated based on the tested EG model and connected as shown in Fig. 4. The schematic diagram for the chain of oscillators is shown in Fig. 5. The output activities from the excitatory cells are recorded and illustrated in Fig. 6. No wrap around is used for the chain simulation. The  $i$ -th oscillator in the chain was governed by (1) and (2), where  $x$  and  $y$  were replaced by  $x_i$  and  $y_i$  respectively. Nearest neighbor coupling was implemented by adding the term (3) to the excitatory equation (1).

$$\xi \alpha (B - x_i) \left\{ \sum_{k \in N_i} [x_k - \Gamma]^+ \right\} \quad (3)$$

Where (3) governs the Excitatory coupling term,  $\alpha$  is the Excitatory coupling strength,  $N(i)$  is the set of the adjacent oscillators that connect to oscillator  $i$  and  $\zeta$  is a scaling factor calculated according to the number of connected neighbors. Oscillators 2 and 3 are connected to their two nearest neighbors as illustrated in Fig. 4, and oscillators 1, 4 are only connected to one neighbor.

In this paper, relaxation oscillations are used. It was shown before that, For a broad range of initial conditions and coupling strengths, arrays of relaxation oscillators synchronized more rapidly than did arrays of sinusoid oscillators under the same coupling and initial conditions [12]. The absolute rates of synchronization however

depend on the coupling strength. At a moderate coupling strength (e.g.  $\alpha=0.10$ ), the model used relaxation, nearest neighbor networks almost always approached synchrony in few cycles, while it was uncommon for their sinusoid counterparts under the same conditions to approach this synchrony criterion in less than 30-40 cycles.

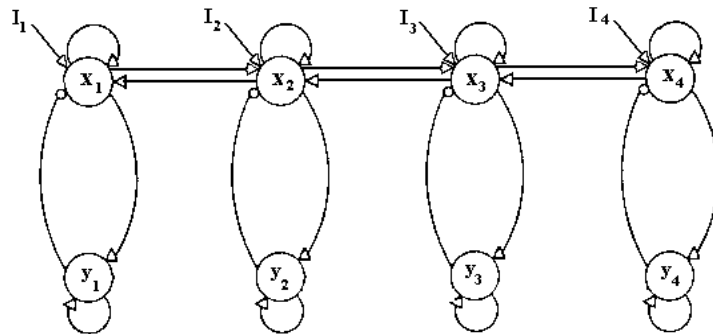


Fig. 4. A diagram showing the interactions between the excitatory and inhibitory units in a chain of four oscillators.

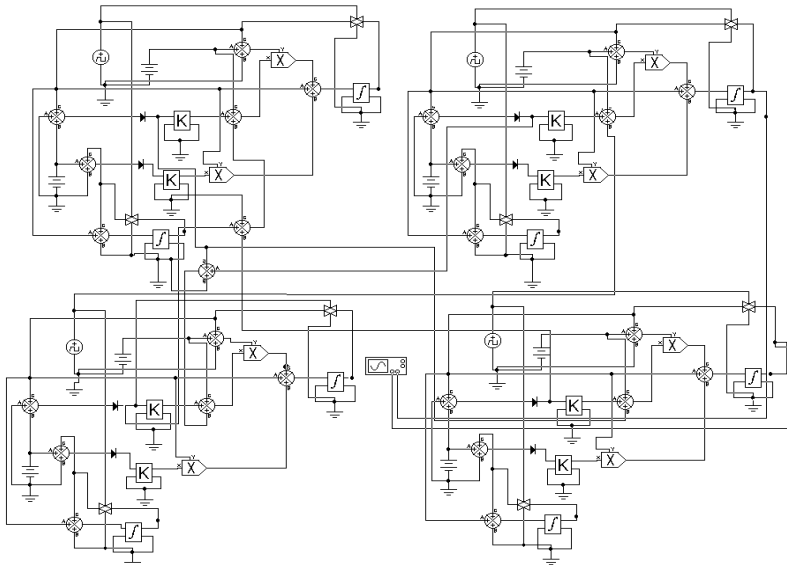
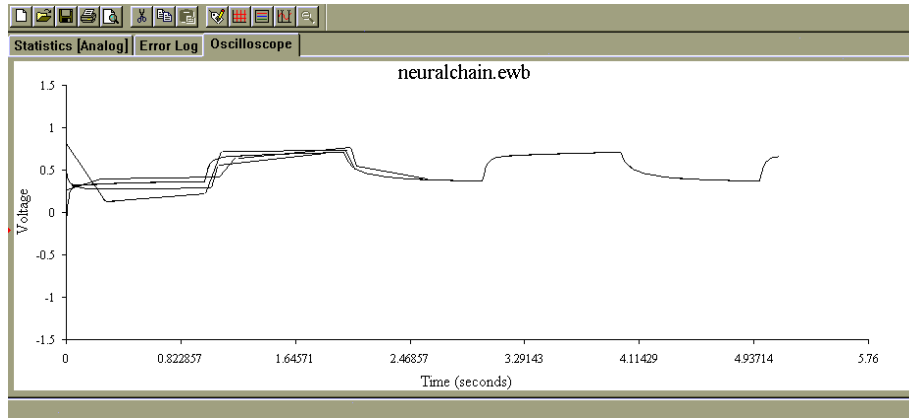


Fig. 5. Schematic diagrams for a chain of four coupled oscillators.



**Fig. 6. The chain output, showing the four neurons synchronization.**

## 5. The Effect of Noise

Oscillatory behavior in neural networks is attracting increased attention, partly due to recent results suggesting that responses are present in visual cortex and may contribute to the solution of the binding problem [13 - 16].

From a theoretical point of view it needs to be clarified in how fast synchronization of oscillations is possible, and what the underlying mechanisms necessary to obtain fast synchronization are. In this section we study the fast-slow oscillator, characterized by the equations (4), and (5), in response to deterministic and noisy inputs. The input consists of two parts: the deterministic input  $S$  and the random noise  $\rho$  that obeys a uniform distribution. We confirm that oscillations occur only if the input lies within a particular interval. We establish that the input has a notable influence on amplitude, frequency and area of oscillations. We find that the behavior of the oscillator is stable in the presence of noise. Armed with these results we study how two competing oscillators (encoding different orientations) interact, how competition affects their amplitude, frequency, area, and the influence of noise.

$$\frac{dx}{dt} = -Ax + (B - x) \{C[x - \Gamma]^+ + S + I + \rho\} - Dx[y - \Gamma]^+ \quad (4)$$

$$\frac{dy}{dt} = E(x - y) \quad (5)$$

In the presence of noise, oscillations occur when the deterministic input  $S$  lies within an oscillation interval that is shifted by roughly the mean of the added noise  $\rho$ . At very high noise intensities, the fast-slow oscillator produces spikes at irregular intervals. In this work, only regularly spaced spikes are countered as oscillations.

The presence of noise hardly affects the slow node, since  $y$  averages the activity  $x$  from the fast node, thus establishing a baseline of activity. Hence, for stronger noise, inhibition from the slow node is bigger, thus ensuring that the fast node remains centered at the same activity, although the amplitude of oscillations varies. Fig. 7 shows the output when connecting two oscillators in a chain and applying input with large amount of noise that acceded 2.7 times of the input value.

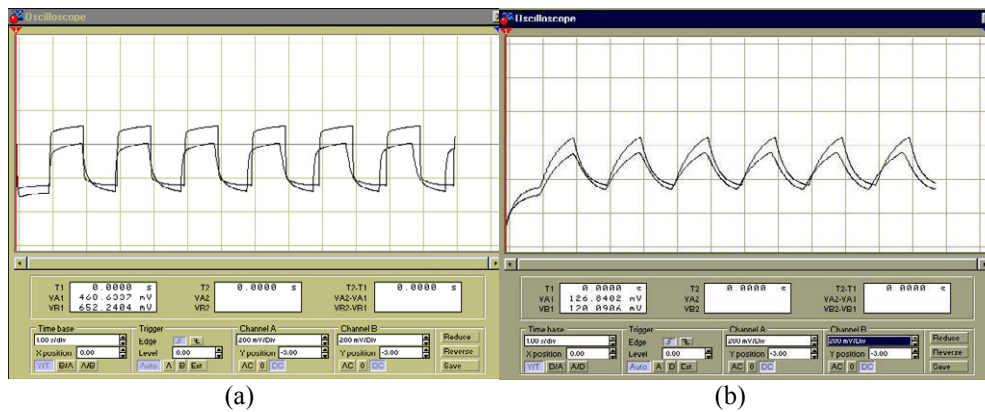


Fig.7. (a) and (b) Traces show the excitatory and the inhibitory outputs respectively of two competing oscillators according to an application of noisy input.

## 6. Human Bimanual Coordination Effects

In the Yamanishi et al [4] finger-tapping task, subjects were required to bimanually tap keys in time to visual signals. The timing signal was varied across ten relative phases: (0.0, 0.1, 0.2...1.0), where  $0.0 = 0^\circ$  and  $1 = 360^\circ$ . The authors observed two properties in the responses of their subjects. First, the subjects 'fingers' tended to slip from intermediate relative phase relationships toward purely in-phase (0.0 and 1.0) or anti-phase (0.5) relationships. Second, the observed in-phase and anti-phase oscillations exhibited less variability than oscillations with intermediate phase relationships. That is, when the subjects were asked to synchronize to signals whose phase relationships varied from 0.0 to 1.0 the standard deviation of the errors was lowest when the phase relationship was near in-phase (0.0 and 1.0 or  $0^\circ$  and  $360^\circ$ ) or pure anti-



phase ( $0.5$  or  $180^\circ$ ). The standard deviations of the errors increased as the subjects were required to move away from the in-phase or pure anti-phase oscillations.

Grossberg [4] described an experimental task in bimanual coordination, which involved moving fingers or limbs in in-phase or anti-phase oscillations. For example, adduction of the right index finger simultaneously with abduction of the left index finger is an anti-phase movement. Concurrent abduction (or adduction) of both fingers is an in-phase movement. A metronome signaled the rate of movement of the fingers. Tuller and Kelso [13] summarize the following four qualitative behaviors found in the bimanual tasks: (1) If a subject was asked to produce a  $180^\circ$  anti-phase oscillation, the subject could do so at low frequencies. But as frequency was increased, the subject eventually switched to an in-phase oscillation. (2) When instructed to perform an in-phase oscillation, the subject could do so at both low and high frequencies. (3) Fluctuations, in which no clear phase relationship dominates, occur before the transition from anti-phase to in-phase oscillations. There does not appear to be clear transition point between ranges of frequencies where only in-phase output occurs and the lower frequencies were both anti-phase and in-phase frequencies occur. (4) Subject's phase errors were minimal at required phases of  $0^\circ$ ,  $180^\circ$ , and  $360^\circ$  (the "seagull effect" described above).

### 6.1. Model of two-channel neural pattern generator

The model is a version of the cooperative-competitive nonlinear feedback network introduced by Ellias and Grossberg [1]. The two-channel pattern generator, briefly summarized in Cohen, Grossberg, and Pribe [4] is depicted in Fig. 8 and obeys the equations:

$$\frac{dx_1}{dt} = -Ax_1 + (B - x_1)[f(x_1) + I_1] - (c + x_1)[D_{11}g(y_1) + D_{12}g(y_2)] \quad (6)$$

$$\frac{dy_1}{dt} = E[(1 - y_1)[x_1]^+ - y_1] \quad (7)$$

$$\frac{dx_2}{dt} = -Ax_2 + (B - x_2)[f(x_2) + I_2] - (c + x_2)[D_{21}g(y_1) + D_{22}g(y_2)] \quad (8)$$

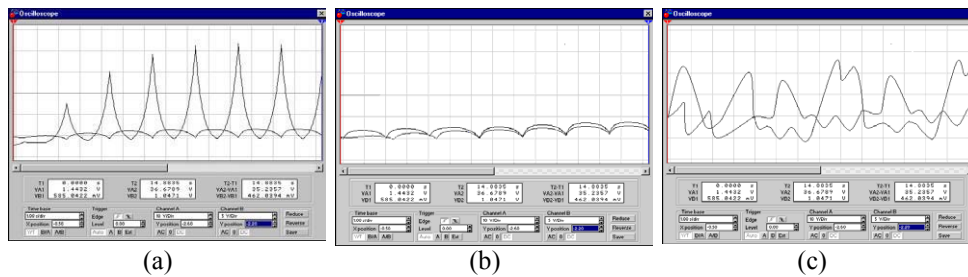
$$\frac{dy_2}{dt} = E[(1 - y_2)[x_2]^+ - y_2] \quad (9)$$

where  $D_{12}=D_{21}$ ,  $[w]^+ = \max(w, 0)$ , and

$$f(w) = \frac{F_1 ([w]^+)^2}{F_2 + ([w]^+)^2}, \quad g(w) = \frac{G_1 ([w]^+)^2}{G_2 + ([w]^+)^2} \quad (10)$$



four effects in response to input pulses,  $I_i$ , that mimic experimental conditions: (1) Increasing the frequency of anti-phase square wave, input caused a bifurcation from anti-phase (Fig. 10.a) to in-phase (Fig. 10.b) oscillations. (2) There was no reverse transition in response to in-phase inputs. (3) Phase fluctuations were exhibited in between the anti-phase and in-phase regimes (Fig. 10.c). (4) The “seagull” effect was observed (Fig. 10.a). The tendency to slip from intermediate phase relationships toward purely in-phase and anti-phase. These results support the hypothesis that oscillations during human bimanual coordination are emergent properties of a neural network whose finger commands compete via slow inhibitory feed back interactions, excite themselves via fast excitatory interactions, and are nonlinearly coupled to shunting membrane processes via nonlinear sigmoid signals.



**Fig.10. Bifurcation from anti-phase to in-phase oscillation in response to anti-phase inputs of increasing frequency. (a) The Low frequency (0.1 pulses per unit time) of the anti-phase inputs  $I_i$  give rise to the anti-phase oscillation. (b) The high frequency (0.85 pulses per unit time) of the anti-phase inputs  $I_i$  give rise to the anti-phase oscillation. (c) The intermediate frequency (0.45 pulses per unit time) of the anti-phase inputs  $I_i$  give rise to the anti-phase oscillation.  $A=1.0$ ,  $B=1.1$ ,  $C=2.5$ ,  $D_{ii}=0.8$ ,  $D_{ij}=0.45$ ,  $E=1.0$ ,  $F_1=9.0$ ,  $G_1=3.9$ ,  $F_2=0.5$ ,  $G_2=0.5$ , The input,  $I_i$ , when on was 0.4 and when off was 0.0.**

## 7. Conclusion

The neural analog presented in this work incorporates most of the important features of the actual biological neurons. One important advantage of this neural analog is that the outputs at the various stages of the analog are processed typically to that of an actual neural data for the direct comparison of the two. This model is flexible in its operation and studies can be carried out on the electrical characteristics of the different kinds of nerve cells by the simple process of changing the required parameters of the circuit.

Since the low cost integrated chips are used in the present model, it serves as a valuable aid to study the single nerve cells as well as the neural nets incorporating

several neurons. The model can be updated in the future as more neural data become available.

Since it is possible to control the amplitudes of the EPSPs and IPSPs and also their frequencies, this circuit will serve as a good aid to study the interactions of EPSPs and IPSPs of various amplitudes and frequency. Since it is possible to set different threshold values by controlling the threshold battery value, this circuit serves as a good means to study the effects of the synaptic inputs at several areas of the postsynaptic membrane having non-uniform distribution of excitability. It is seen that action potential and synaptic potentials are transmitted without any distortion with active axon model as in neurons in living creatures.

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## تصميم نموذج محاكاة إلكتروني لمذبذبات جروسبرج

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(قدّم للنشر في ١٠/١٠/١٩٩٩م؛ وقبل للنشر في ١٤/٠٩/١٩٩٩م)

**ملخص البحث.** سعى عدد من الباحثين، بعد نجاح تجارب المحاكاة الحاسوبية، إلى تنفيذ النماذج الناجحة للخلايا العصبية على دوائر مدججة، لما لها من المميزات من حيث: صغر الحجم، و قلة التكلفة، و سهولة التداول، إلى جانب تسهيل ملاحظة واكتشاف سلوك الخلايا العصبية بطرائق ملموسة باستخدام عارضة الموجات أو شاشة العرض الحاسوبية. ونعرض في هذه الورقة لمحاكاة تصميم إلكتروني للخلية العصبية طبقاً لنموذج جروسبرج للمذبذبات العصبية المتعاقبة مع نتائج تؤكد نجاح هذا النموذج في محاكاة جميع وظائف الخلية.

ولقد تم توصيل عدد من وحدات نموذج الخلية على شكل سلسلة مترابطة، وجاءت نتائج التجربة مطابقة لنظيرتها باستخدام المحاكاة الحاسوبية. ويعد هذا النموذج خطوة مهمة في اتجاه بناء وتطوير خلية عصبية اصطناعية، مما يسهم في تطوير نظم الذكاء الاصطناعي التي يتسارع معدل استخدامها في تطبيقات شتى في عالمنا المعاصر.