

Contents

Foreword	v
Preface	vii
1 Dimensions and units	1
1.1 Fundamental concepts	1
1.1.1 Base units and dimensions	1
1.1.2 Dimensions of common physical quantities	2
1.1.3 The Buckingham Pi theorem.....	3
1.1.4 Absolute errors, relative errors, and units	5
1.1.5 Units and computers.....	5
1.1.6 Unit systems.....	5
1.1.7 Example on challenges arising from unit systems	6
1.1.8 PhysicalQuantity: a tool for computing with units	7
1.2 Parampool: user interfaces with automatic unit conversion ...	9
1.2.1 Pool of parameters	10
1.2.2 Fetching pool data for computing.....	11
1.2.3 Reading command-line options	11
1.2.4 Setting default values in a file	12
1.2.5 Specifying multiple values of input parameters	13
1.2.6 Generating a graphical user interface.....	14
2 Ordinary differential equation models.....	17
2.1 Exponential decay problems	17
2.1.1 Fundamental ideas of scaling.....	17
2.1.2 The basic model problem	18
2.1.3 The technical steps of the scaling procedure	19
2.1.4 Making software for utilizing the scaled model	21
2.1.5 Scaling a generalized problem	25
2.1.6 Variable coefficients.....	31
2.1.7 Scaling a cooling problem with constant temperature in the surroundings	32

2.1.8	Scaling a cooling problem with time-dependent surroundings	33
2.1.9	Scaling a nonlinear ODE	37
2.1.10	SIR ODE system for spreading of diseases	39
2.1.11	SIRV model with finite immunity	41
2.1.12	Michaelis-Menten kinetics for biochemical reactions	42
2.2	Vibration problems	49
2.2.1	Undamped vibrations without forcing	49
2.2.2	Undamped vibrations with constant forcing	53
2.2.3	Undamped vibrations with time-dependent forcing	53
2.2.4	Damped vibrations with forcing	61
2.2.5	Oscillating electric circuits	67
3	Basic partial differential equation models	69
3.1	The wave equation	69
3.1.1	Homogeneous Dirichlet conditions in 1D	69
3.1.2	Implementation of the scaled wave equation	71
3.1.3	Time-dependent Dirichlet condition	72
3.1.4	Velocity initial condition	75
3.1.5	Variable wave velocity and forcing	77
3.1.6	Damped wave equation	80
3.1.7	A three-dimensional wave equation problem	81
3.2	The diffusion equation	81
3.2.1	Homogeneous 1D diffusion equation	82
3.2.2	Generalized diffusion PDE	83
3.2.3	Jump boundary condition	85
3.2.4	Oscillating Dirichlet condition	86
3.3	Reaction-diffusion equations	89
3.3.1	Fisher's equation	89
3.3.2	Nonlinear reaction-diffusion PDE	91
3.4	The convection-diffusion equation	92
3.4.1	Convection-diffusion without a force term	92
3.4.2	Stationary PDE	95
3.4.3	Convection-diffusion with a source term	97
4	Advanced partial differential equation models	99
4.1	The equations of linear elasticity	99
4.1.1	The general time-dependent elasticity problem	99
4.1.2	Dimensionless stress tensor	101
4.1.3	When can the acceleration term be neglected?	101
4.1.4	The stationary elasticity problem	103
4.1.5	Quasi-static thermo-elasticity	105
4.2	The Navier-Stokes equations	106
4.2.1	The momentum equation without body forces	107
4.2.2	Scaling of time for low Reynolds numbers	109

Contents	xiii
4.2.3 Shear stress as pressure scale	110
4.2.4 Gravity force and the Froude number	110
4.2.5 Oscillating boundary conditions and the Strouhal number	110
4.2.6 Cavitation and the Euler number	111
4.2.7 Free surface conditions and the Weber number	112
4.3 Thermal convection	113
4.3.1 Forced convection	113
4.3.2 Free convection	114
4.3.3 The Grashof, Prandtl, and Eckert numbers	117
4.3.4 Heat transfer at boundaries and the Nusselt and Biot numbers	120
4.4 Compressible gas dynamics	121
4.4.1 The Euler equations of gas dynamics.....	121
4.4.2 General isentropic flow	123
4.4.3 The acoustic approximation for sound waves	124
4.5 Water surface waves driven by gravity	126
4.5.1 The mathematical model	126
4.5.2 Scaling	127
4.5.3 Waves in deep water	128
4.5.4 Long waves in shallow water	129
4.6 Two-phase porous media flow	130
References	135
Index	137