Chapter 10
Tensions in Teaching Mathematics in Contexts of Language Diversity

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10.1 Introduction

It is apparent from existing research that teaching and learning mathematics in contexts of language diversity presents learners, teachers, and policymakers with challenges. These challenges take many forms, but all can be thought of in terms of tensions. By "tension," we mean a challenging situation with no clear-cut solution, a situation in which competing influences or forces suggest different, often opposing courses of action. In this chapter, we present four case studies of such tensions arising in mathematics classrooms in Canada, Malaysia, and South Africa. One of our aims is to show how similar tensions arise in quite different contexts. A second aim is to examine the role of teachers in mediating these tensions. To do so, we draw on aspects of the theory of language developed by Bakhtin (1981). From this perspective, language use always results in particular kinds of tension, as we explain later in the chapter.

The chapter is organised into four main sections. First, we summarise some of the tensions that have already been reported in the literature on teaching and learning mathematics in contexts of language diversity. Next, we introduce the theoretical perspective on language, drawing on Bakhtin's work. We then present our four case studies. Finally, we discuss the similarities and differences arising across the cases and consider some of the implications of this work.

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10.2 Tensions in Mathematics Classrooms in Contexts of Language Diversity

While the term "tension" has not been widely used, similar words arise frequently in the literature (see Barwell, 2009, 2012). Most notably, Adler (2001) wrote of the different 'teaching dilemmas' facing mathematics teachers in multilingual South Africa. For Adler, teaching dilemmas were situations in which teachers perceived a choice of actions, and each one presented both advantages and disadvantages. For example, Adler referred to the dilemma of transparency to describe a situation described by teachers in which they felt that they had to decide whether mathematical language should be made visible through explicit attention, or whether it should be left as a transparent medium for mathematical discussion. A similar tension between informal language and mathematical language has been observed in different forms in several other studies in different parts of the world, including in the United States (e.g., Khisty, 1995; Moschkovich 2008), South Africa (Setati & Adler, 2000), Canada (Barwell, 2014), and Australia (Clarkson, 2009). As implied by our definition of tension, there is no neat resolution to the tension between informal and mathematical language. Teaching and insisting on highly formalised mathematical language in contexts of language diversity will simply disenfranchise many learners. On the other hand, not to provide them with the opportunity to learn more formal mathematical language will, in the long run, also disenfranchise them, even if they have a good understanding of mathematics. Again, teachers' responses to these kinds of situations will have an impact on students' participation in mathematics.

A second widespread tension arises between the language of instruction for mathematics in school, and the languages used by students outside of school. This tension is apparent in many other studies in different parts of the world, including Australia (Clarkson, 2007; Clarkson & Dawe, 1997; Parvanehnezhad & Clarkson, 2008), Malawi (Chitera, 2009), Malta (Farrugia, 2009), Pakistan (Halai, 2009), Swaziland (Dlamini, 2008), Catalonia, Spain (Planas & Setati, 2009), and Malaysia (Lim & Ellerton, 2009). It is also worth remembering that in many contexts, code-switching is *not* used, despite at least some students using two or more languages in their daily life outside school (Setati & Barwell, 2006). In each of these studies, there is a tension between the languages students use in their daily life and the language used to teach and learn mathematics. There is no simple way out of this tension. Students in most parts of the world live with language diversity: many languages are spoken outside of school, even if a single language is used for teaching and learning mathematics. It is clear that teachers play a key role in navigating this tension.

Finally, Setati's (e.g. 2008) work has highlighted a third significant tension arising in language-diverse mathematics classrooms. This tension derives from the status that different languages or ways of talking have in wider society. Specifically, there is a tension between the desirability of English in South African society and the facility with which learners are able to make sense of mathematics taught in English. Similarly, in work conducted in the United States and in Catalonia, Spain, Planas and Civil (2013) show how the societal value of different languages plays an

important mediating role in mathematics classroom participation. They make the point that this mediation can undermine approaches to teaching mathematics that in other respects are open to the various linguistic resources that students may bring.

Our summary highlights some common tensions that have been reported in different mathematics classrooms around the world. We offer three observations about this work. First, the fact that each of them has been reported in several contexts suggests that they are likely to be widespread. There is no reason to suppose that a tension around the use of formal and informal language in multilingual mathematics classrooms is restricted to the places where research has been conducted.

Second, it is also apparent that the above tensions are all interrelated. The use of code-switching in mathematics classrooms, for example, is related to the policy and preference for using high-status languages rather than languages that students use at home. Similarly, the tension around the use of informal mathematical language is accentuated in contexts of language diversity, when students have less confidence or proficiency in using the official language of schooling. It is also clear that teachers find themselves in a pivotal position in dealing with these tensions in their practice. The choices teachers make and the strategies teachers use will have a potentially significant impact on their students' opportunities to engage in mathematical thinking and to learn mathematics. A teacher who enforces a one-language-only policy, for example, will empower some students in her class (those with the cultural resources to support use of that language) and disempower others. A teacher who encourages multiple language use in mathematics will also empower some students and disempower others, but not in the same way as in the first situation.

Third, the research we have summarised has been valuable in describing these tensions but has not sufficiently developed a theoretical perspective that accounts for or explains them. This is not to say that this work has not drawn on robust theoretical frameworks. Adler (2001) drew on a Vygostkian perspective to show how competing pedagogical priorities lead to teachers experiencing dilemmas. Setati's (2005) work is framed by Gee's (1999) theory of discourse and cultural models, through which she was able to show how individuals often held competing cultural models about language or mathematics. What this work has done less well, however, is to develop a theoretical account that explains how these tensions arise. Such a perspective would allow for a deeper understanding of the nature of the many tensions that arise in different contexts and would suggest new ways to tackle them. In the next section, we show how Bakhtin's theory of language provides such a perspective.

10.3 Bakhtin's Theory of Language

Bakhtin was predominantly a literary theorist with an interest in, among other things, the nature of language used in novels or in poetry. Embedded in his work, however, is a theory of language that challenges many common assumptions. In particular, Bakhtin's work challenges the ideas that language can be codified and categorised in a simple way, or that words or definitions are the basic units of a

language. For Bakhtin (1981), language lives in interaction; he argued that the basic unit of language is, therefore, the utterance, situated within interaction. Moreover, the words that we all use when we speak or write are always derived from our prior experience of these words and of language. We are always in some sense borrowing or recycling words we first heard or read in other people's utterances. A key feature of Bakhtin's view of language is that he does not privilege a formal model of language purity, nor does he privilege a use-based model of language diversity. Instead, he proposes that both are present in the form of opposing forces.

Bakhtin (1981) used the term unitary language to refer to a view of language as coherent and unified. Unitary language "gives expression to forces working toward concrete verbal and ideological unification and centralization, which develop in vital connection with the processes of sociopolitical and cultural centralization" (p. 271). Implicit in this idea is the link between unitary language and political projects of nationalism or colonialism, as well as ideas about language purity or correctness. Bakhtin argues that the idea of unitary language exerts a constant pressure on language use, for which he uses the metaphor of centripetal force. Centripetal forces seek to impose or coerce language and its speakers into idealised standard ways of talking. Such forces are familiar in education: the preference for a single language of instruction; the teaching of the rules of grammar; the correction or suppression of non-standard ways of speaking or writing.

Bakhtin (1981) also stresses the diversity apparent in language in use, highlighting "the social diversity of speech types" (p. 263), translated as heteroglossia, to encompass the different ways of talking apparent in different social groups, political groups, regional dialects, and accents, even of families or individuals or particular times. Heteroglossia is apparent in any classroom, in which many different intersecting social languages can be identified: the languages of curriculum, of subject matter, of pedagogy, of teachers, of students, of school, of home, of working class or middle class, of neighbourhoods, and of age groups, and so on. Bakhtin uses the metaphor of centrifugal force to characterise the pressure that this diversity exerts.

These two ideas of language—unitary language and heteroglossia—are in a constant struggle analogous to the tension between centripetal and centrifugal forces. Centripetal forces seek to impose or coerce language and its speakers into idealised standard ways of talking, while centrifugal forces are apparent in the multivocal, multilingual, hybrid ways in which language is actually used. The tension between the centripetal force of unitary language and the centrifugal force of heteroglossia is present each time we speak and shapes what we say. Moreover, this tension is inherent in language. Without a degree of uniformity, language would be meaningless; language needs patterns and rules to make communication possible. At the same time, without diversity self-expression would be impossible. As Duranti (1998) points out, this tension is related to issues of power and marginalisation. The unitary forces tend to promote versions of language that correspond to the language of the powerful.

It is apparent that the tensions that arise in teaching and learning mathematics in contexts of language diversity can all be understood as reflecting the tension between centripetal and centrifugal forces. The tension between formal and informal language in mathematics classrooms is precisely a tension between more unified stan-

dardised forms of mathematical expression (i.e. unitary language) and more diverse, idiosyncratic expressions of mathematical meaning. The tensions around code-switching arise from a centralising tendency to demand that a single language be used for learning and teaching mathematics and the need to use a wider range of forms of expression to make mathematics meaningful. And Setati's (2008) research highlights the tension between the unifying movement to prefer English as a high-status international language and the multilingual reality of South African society. These forces, moreover, are not a simple case of government policy vs. the people. The pressure for English in South African schools comes from parents; official policy permits instruction in any of South Africa's official languages.

This theoretical perspective has the potential to extend the work summarised in the previous section. In particular, Bakhtin's theory of tensions explains the widespread prevalence of various specific tensions in the literature on mathematics teaching and learning in contexts of language diversity. They are widespread because, in a general sense, language is always filled with such tensions. We can thus clarify the nature of the teaching dilemmas described by Adler (2001): these dilemmas arise when the tensions that are present in language lead to teaching situations in which teachers are faced with competing or conflicting choices. Similarly, the competing cultural models in Setati's (2005) work reflect the tension between centripetal forces (English is valuable) and centrifugal forces (multilingualism is valuable).

To explore the role of centripetal and centrifugal forces in mathematics classrooms in contexts of language diversity, in the rest of the chapter we focus on four cases drawn from different settings in different parts of the world. Our particular focus is on the role of mathematics teachers in mediating the tensions that arise in each case. The four cases comprise: (1) a multilingual mathematics classroom in South Africa, (2) the use of an international language to teach mathematics in Malaysia, (3) teaching and learning mathematics in a class for new immigrants in Canada, and (4) immigrant students and teacher in a mathematics classroom in South Africa.

10.4 Case 1: Mathematics in a Multilingual Mathematics Classroom in South Africa

Language diversity is one of the distinctive features of South Africa. The country's eleven official languages include nine African languages. The policy environment allows schools and learners to select their language of instruction and encourages multilingualism and practices such as code-switching. Despite this seemingly progressive policy environment, research shows that schools in South Africa are not opting to use learners' home languages as the language of instruction, in both policy and practice and thus English remains the preferred language of instruction (see, for example Setati, 2008; Setati & Adler, 2000; Setati, Adler, Reed, & Bapoo, 2002). While African languages are spoken widely, using them as language of instruction is still associated with apartheid education and hence with inferior education.

This case[1] involves a multilingual Grade 4 mathematics class of 8–14 years old in a school located in an African township, west of Johannesburg. The learners in the school are all African. The chosen language of instruction in the school is English. The learners share a main language, which is Setswana and most of them can speak, read, write or understand another two or more African languages. The learners also study three languages as subjects—Setswana, English, and Afrikaans. At the time of data collection the teacher could speak six languages and shared a main language, Setswana, with her learners.

During an interview with the teacher, some of her remarks gave a sense of the centripetal forces that she experienced:

Teacher: If we changed our [mathematics] textbooks into Setswana and set our exams in Setswana, then my school will be empty because our parents now believe in English.

It became clear that she was struggling to emphasise the unitary role of language despite the local realities that have to do with the learners' level of proficiency in English. On the one hand, she expressed a view that learners should be taught in their home languages so that they can understand the mathematics that they are being taught. On the other hand, she expressed the need for her learners to develop fluency in English so that they are able to communicate with people from "other schools or cultures":

Teacher: The child must learn with the language that he or she will understand … the child must understand what you are teaching him … But still you must not confine a child to a situation where she won't be able to understand other people when she meets or where he meets other people from other schools or other cultures … it is said that English is an international language …

The teacher is attempting to ensure that her learners are initiated into the idealised way of communicating in English, the language of power. While the teacher wants her learners to understand mathematics, she also wants them to learn and practice speaking English so that they can fit into the idealised standard ways of talking in a world in which English dominates. While she indicated that during teaching she uses both English and Setswana, it seems that she uses Setswana to support not only learner access to mathematics, but also improve their fluency in English. There are pragmatic reasons for encouraging English. In the following extract the teacher explains them:

Teacher: I encourage them to use English … The textbooks are written in English the question papers are in English. So [if you use their language] you find that the child doesn't understand what is written there because all the time you encourage them to speak in Setswana and then you give him the question that has been written in English … Like for instance let me make an example last year during exam time we had a problem children asking

[1] A detailed analysis of this example can be found in Planas and Setati Phakeng (2014).

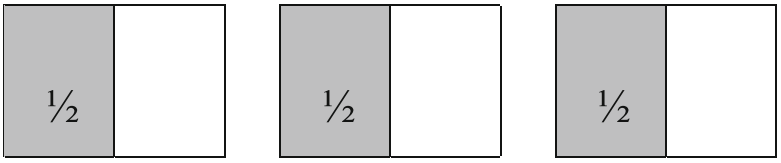
raising their hands asking the invigilator all the questions that they do not understand and the problem was the language not the question itself.

The teacher's emphasis on the use of English conforms to a unitary language perspective. While she seems to recognise its limitations she seems to be constrained by the centripetal forces related to the idea of English as an international language. All she can do is to prepare the learners for participation in this international world, and teaching mathematics in English is an important part of this preparation.

Observation of the teacher during a lesson on fractions (halves and quarters) underlined the role of centrifugal forces. The teacher introduced the lesson using paper cutting and then moved on to using pictures and stories. She did not use the words denominator and numerator; she spoke about the number on top and the number below. She focussed more on getting children to understand what the numbers represent rather than memorising the terms numerator and denominator. In most of the exercises given, learners had to make a pictorial representation of given fractions. Heteroglossia is evident in the switching between languages (Setswana and English), discourses ('academic' and 'everyday' mathematical discourses) and mathematical representations (diagrammatic and symbolic). In the extract below the teacher challenged her learners to imagine themselves at Mr. Nkomo's store buying bread.

Teacher: I want you to think. Close your eyes, and think about the shopkeeper. What will Mr. Nkomo do when you say, "Mr. Nkomo I want three halves." Anyone? (One of the learners goes to the board to draw the bread)

Teacher: So Bernard is showing us how many halves.



Learners: Three halves.

Teacher: Bernard is showing us three halves. (Pointing at the each of the pieces labelled ½) So this is one half, one half and another one half, neh!

Learners: Yes.

While Bernard's diagrammatic representation is mathematically correct, it seems that the teacher was not satisfied with it and therefore engaged learners in a discussion about a realistic situation of a shopkeeper selling bread. She then challenged the learners by asking them to think about how Mr. Nkomo should handle this problem in an everyday context and continued to give her representation of 3/2:

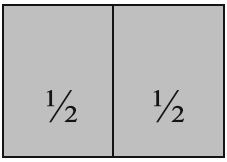
Teacher: When you go to the shop and you say you want three halves, why does not Mr. Nkomo just take one bread and say hey I don't want to waste my bread. I will just give you one bread and I will cut it into three equal parts. Why does he not do that, Molefe? Why Mr. Nkomo a sa nke borotho bo boiwane, a be a bokgaola dipiece tse three [why doesn't Mr. Nkomo take one loaf and cut it into three parts] and say I

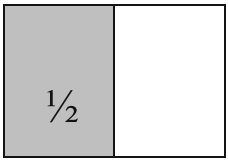
can't take another bread because you are going to waste my bread. Why a sa etsa yalo? Why does not he do that? Why does he not take one bread and cut it into two equal halves and another into three equal pieces, S'bongile?

S'bongile: Because they are three two.

S'bongile's response is based on the symbolic representation of three halves, which is 3/2, hence she says "three two". The denominator "2" indicates the number of pieces the whole is divided into. The teacher, however, missed this subtle switch in mathematical representation and continued to request for more responses.

Teacher: Why a sa nke borotho boiwane a be a bo kgaola piece tse three tse lekanang? [Why doesn't he take one loaf and cut it into three equal parts?] Let's say Mr. Nkomo takes one bread and he cuts it into two equal parts. (She draws the diagrams below on the board.) He gives the first child the one half and the second child one half. But there were three children. Mr. Nkomo then takes another bread and cuts it into two equal parts and gives another child his half. Why does he not do that, Victor?





Victor: Because they are not equal.

Teacher: Are because they are not equal. Are these parts not equal? Are they equal?

Victor: Yes.

The centrifugal forces evident in the interaction above arise not only as a result of the use of multiple representations but also multiple discourses. While the learners provided the teacher with a correct mathematical representation of three halves, the teacher was expecting them to think of the problem as it may be dealt with in real life. The teacher thus expected the learners to think about the fact that Mr. Nkomo does not want to have three half loaves of bread left over because that can be wasteful if nobody else buys them. So while the learners were engaging in academic mathematical discourse, the teacher was also expecting them to also engage in everyday mathematical discourse. The teacher mediates between these discourses, in this case preferring an everyday discourse.

10.5 Case 2: Using an International Language to Teach Mathematics in Malaysia

In Malaysia, English is not the first language of the majority of Malaysian teachers and pupils. However, in 2003, the Malaysian Ministry of Education took the step of implementing the policy of teaching mathematics and science in English. In fact, almost all teachers who are presently below 45 years old have undergone their entire education with languages other than English as the language of instruction. English was only taught as a compulsory subject before 2003. Teaching mathematics in English poses great challenges and tensions, particularly for younger mathematics teachers. After much controversy the policy was reversed in 2012. However, the debates about which language to use continue.

For this case, we draw on data from one Malaysian Chinese primary school (reported in Lim & Presmeg, 2010), from the period in which the above-mentioned language policy was still being implemented. There are three types of primary schools in Malaysia, differentiated by the medium of instruction: national primary schools [SK] (Malay as medium of instruction); Chinese primary schools [SJKC] (Mandarin) and Tamil primary schools [SJKT] (Tamil). In the Chinese primary school described in this case, Mandarin was the most used language both in school and at home for the majority of the teachers and students. Mathematics was taught in Mandarin from Grade 1. However, due to the complex demands of the Malaysian Chinese community mathematics was actually taught bilingually in Mandarin (the pupils' mother tongue) and English.

As in the previous case, in Malaysia, English is highly valued as the language of power, particularly in meeting the challenges of globalisation, and particularly in mathematics, science and technology. In view of the reported decline in Malaysian students' English language proficiency, and the belief that "teaching the subjects in the science disciplines in English would expedite acquisition of scientific knowledge in order to develop a scientifically literate nation by the year 2020" the government introduced the new language policy for mathematics and science education (Choong, 2004, p. 2). However, the implementation of this policy drew much criticism and debate. Those who were against the policy argued that the teaching of mathematics and science in English would not help to rescue the deteriorating standard of English, whereas the proponents claimed that making English a tool of learning is the most effective way of ensuring students are proficient in English as well as upgrading students' achievement in mathematics and science.

For the Chinese school community, strong opposition to the policy was apparent from the beginning, particularly from influential Chinese education groups in Malaysia. These groups opposed the teaching of mathematics and science in English for fear of increasing the burden on school children and changing the distinctive character of Chinese schools. They also argued that mathematics is best taught in students' mother tongue, and were proud that research had shown that students in Chinese schools (SJKC) consistently achieved better than their counterparts in SK and SJKT schools. Hence, after much political negotiation, mathematics was taught

as two subjects in Chinese primary schools: mathematics in Mandarin (six periods) and mathematics in English (two periods). These policy initiatives are characteristic of centripetal forces since they seek to promote a particular language (English). In this case, the centralising tendencies of language policy led to a clash between two relatively powerful languages, English and Mandarin.

Analysis of mathematics classroom interaction revealed a difference in the percentage of English language used in high-attaining classes compared with low-attaining classes. English was used in more than 97 % of interaction in the high-attaining classes, but in less than 52 % of interaction in low-attaining classes. Where code-switching occurred, it was dominated by the following patterns of interaction, as shown in Table 10.1.

These findings suggest that there is greater language diversity in interaction in the low-attaining classes, at least in terms of the use of Mandarin and English. Moreover, as highlighted in Lim and Presmeg (2010), both the experienced and novice teachers were not able to teach mathematics entirely in English for both high- and low-attaining classes. Particularly in low-attaining classes, teachers said that they used extensive code-switching, in order to allow pupils who are weak in English to catch up with the peers. This way of teaching mathematics in two languages has become a teaching dilemma for the teachers. Ideally, teachers would like to teach mathematics monolingually (in English) as instructed by the newly implemented language policy. Teachers are also worried, however, about whether their students can understand their mathematics lesson fully if it is taught totally in English. The latter concern is greatly related to their confidence in their students' English language proficiency. A substantial amount of teaching time was devoted to translation, especially of the terminology of mathematics.

However, to expedite the teaching, these teachers sometimes opted to teach in the pupils' mother tongue (Mandarin) only. For example, an experienced teacher in the study, Mrs. L, disclosed that when she explained difficult concepts, even for a good class, she opted to use Mandarin sometimes. This is because "我要看到很快的效果的话, 我就用华语" [If I want to see the effect quickly, I will use

Table 10.1 Types of code-switching in Malaysian mathematics classrooms

Types of code-switching	Examples
1. Translation	"Width, 宽度"; "What is the unit you use, to measure the mass of heavy object? 你们用什么单位来测量, 比较重的东西?
2. Translation involving terminology	Teacher: 华语叫什么meter? [What is meter in Mandarin?] Student: 米 [meter]
3. English is used for units and certain common words, such as pencil	"一个cm等于十个mm" [1 cm equal to 10 mm] or "那边量起。量给老师看。来, 第一组, 来, 你去写pencil,你量到多少, 出去写, 快点, 玲伊。" [Measure from that side. Measure to show teacher. Come, first group, come, you go and write pencil, how many have you measured, go and write, quick, Linyi.]

Mandarin]. In order to speed up the lesson and to overcome time constraints, the teacher opted to use Mandarin because pupils were more familiar with this language. This is an example of how teachers mediate the tensions between centripetal and centrifugal forces, in this case between a language policy and the needs of students. It is clear in this case that the mediation of these tensions is intimately linked to concerns about students' mathematical learning (what Setati, 2008, calls 'epsitemological access').

10.6 Case 3: Immigrant Students Learning Mathematics in a Sheltered Language Program

The first two cases concerns classrooms in which all students are assumed to be growing up multilingual. In this third case, from Québec, Canada, the school system assumes that most children will grow up monolingual. Newcomers must adapt to this situation. In Québec, the children of immigrants are required to join the French-medium school system. Many such children do not, on arrival, speak French. These children are placed in a special class, called a *classe d'accueil*, the main goal of which is for the students to learn enough French to join mainstream classes. Students stay in the *classe d'accueil* for up to a year, by which point they are expected to be proficient enough to survive in the mainstream school system.

This case concerns a class of 9–10-year-old students in a *classe d'accueil* in a medium sized city in Quebec. There were 18 students in the class who had come from a variety of backgrounds, including South America, West Africa, India, and the Middle East. Some of the students have lived in a third country before moving to Canada. About half the students in the class speak Spanish; other languages represented include Portuguese, KiSwahili, Hindi and Arabic. The teacher is a White, francophone Québecoise with several years of teaching experience. Although the main focus and purpose of the class is on learning French, the students do devote some time to other curriculum subjects, including mathematics.

The existence of the *classe d'accueil* is a result of the centripetal language forces in the Québec education system. The fact that students are expected to learn French and are provided with a special class, the express purpose of which is to teach students French arises from a unitary ideology that positions French as the main language of education and of society. The origins of this policy are in concerns that French in Québec was or is being eroded by its proximity to the surrounding English-dominant provinces of Canada and the United States and a political project to reinforce the role of French in Québec society. While the defence of French is understandable, it is important to note that there is little role in the *classe d'accueil* for students' expertise in other languages. As the teacher says:

> Le but c'est qu'ils soient capable de communiquer en français assez pour être capable d'apprendre dans une classe régulière une année après […] Nos deux grands axes qu'on doit développer dans notre programme c'est communiquer en français et s'intégrer a son milieu (.) d'accueil. Donc oui on doit faire des mathématiques mais axé sur le français. C'est sur le vocabulaire.

> The goal is that after a year they'll be able to communicate in French enough to be able to be in a mainstream class […] The two main thrusts of the program are communication in French and integration into their new environment. So yes we have to do mathematics but focused on French. On vocabulary.

This general unitary approach to the use of French has an impact on the learning and teaching of mathematics. For example, less time is allocated to mathematics than is the case in mainstream classes for this age group: language policy has a direct impact on the learning of mathematics. Furthermore, much of the emphasis during mathematics lessons tends to be on vocabulary development. The teacher also reported that at the start of the year, when the students could not speak French, the class was very quiet. Hence, during mathematics lessons, there could be little discussion. This changed as the students became more proficient and by the end of the year there was considerable lively discussion.

The heteroglossia in the class is readily apparent. Even when students were only speaking French, they did so with a variety of non-standard accents, reflecting the languages they spoke at home. For example, during a lesson on convex and non-convex shapes, one exchange involved multiple pronunciations of the (French) word angle. The students are sorting a set of regular and irregular shapes:

Teacher: pourquoi tu les mets comme ça [why are you putting them like that?]
S6: c'est plus parce que (…) attends attends (…) [it's because … wait, wait …]
Teacher: s'il vous plait dans deux groupes tu vas me les laisser tantôt [in two groups please, I'll look at them in a minute]
S6: c'est que tout ça ont des angel (…) angle (.) toute ça (.) il y a des angles et [it's that all that have an'gel … angle (.) all that (.) there are angles and]
S7: ^awngles^
S6: ungles et ici (…) deux angles ici [ungles and here … two angles here]

Similarly the use of non-standard forms, including vocabulary, led to frequent repair sequences, in which the participants came to a common understanding. In the following example, a student is justifying his claim that the letter L is a non-polygon:

Teacher: bon (.) L (.) S40 (…) L tu le mets ou? [good, L, S40, where would you put L?]
S40: polygone
Teacher: dans polygone? [in 'polygon']
S40: non non-polygone
Teacher: dans non-polygone (.) pourquoi? [in 'non-polygon' (.) why?]
S40: parce que le carré des (cous) des [because the square…]
Teacher: des cous?
S40: non (…) il y a un (cou) [no (…) there's a (cou)]
Teacher: un trou ok la ligne n'est pas fermée (.) très bien bravo [a hole, ok, the line isn't closed, good, well done]

The student appears to say "cou" (neck) or possibly "coups" (hits or blows) which has the same pronunciation, which the teacher struggles to interpret, eventually revoicing it as "trou" (hole) and praising the student for a correct choice.

On some occasions, students used their home languages, particularly in the case of Spanish-dominant students. The teacher, however, explicitly enforces a norm of students not using their home languages, as arose during an exchange about the meaning of convex, in the same lesson:

S46: euh si on peut espagnole [er, if I can Spanish]
Teacher: non non
S46: el el le convexe les cotés ont une forme des V [el el the convex the sides are in a V]
Teacher: comme un V? [like a V?]
S46: ouais un V mais très [yes a V but very]
Teacher: ok si moi je dessine ça comme ça? [ok if I draw it like that?]
S46: ah
Teacher: il n'y a pas de V [there isn't a V]
S46: oui erm
Teacher: ah si je tourne? [ah if I turn it]
S46: oui

Here, the student asks to use Spanish but is not given permission. His next utterance begins in Spanish, then moves more into French, though with apparent effort and with many accompanying gestures which the teacher interprets verbally. The teacher's role therefore consists of refusing the use of Spanish and of interpreting and voicing the student's efforts.

In the *classe d'accueil*, then, the teacher mediates between a French-only policy and the varieties of French and other languages that the students bring. In the second example, above, the trouble arises because the student is confined to using French, despite struggling to find an appropriate word to justify a mathematical decision (his categorisation of the letter L). The teacher's mediation involves maintaining French as the language for their exchange, as well as seeking to interpret what the student is saying, offering a possible word and hearing his words and gestures as mathematically correct.

10.7 Case 4: Immigrant Students and Teacher in a Mathematics Class in South Africa

Since 1994, South African cities have become primary destinations for migrants from around the continent. As migrants cross borders into South Africa, they bring diverse languages into an already linguistically diverse context. Some of the languages spoken by African migrants are not spoken in mathematics classrooms in South Africa. For example, some of the migrants are teachers and children from

French-speaking countries. These children join mainstream classes where the language of instruction is English. Research has shown that some mathematics teachers in these classrooms use the learners' home languages during the teaching of mathematics to support learners who are in the process of learning English (Adler, 2001; Setati, 2005).

This case concerns a multilingual Grade 11 mathematics class with immigrant learners from the Democratic Republic of Congo (DRC), in a school located in an inner city residential neighbourhood of Johannesburg. The students were learning linear programming. The class had 26 students: 3 from the DRC, 1 each from Malawi and Zimbabwe and the rest from South Africa. Therefore, languages represented that were not official languages in South Africa included Shona, Lingala, and Chichewa. The teacher was from the DRC and has many years of teaching experience, including more than 10 years teaching in South Africa. The teacher shared English with all learners in the class. He also shared French (the language of instruction in the DRC) and Lingala, with the immigrant learners that are a focus in this case.

During an interview, the unitary approach to language was emphasised by the teacher when he stated that everything is taught in English and, therefore, immigrant learners had to learn it. However, in the course of the interview (which was conducted in English) centripetal forces were evident as he recognised that he sometimes switches to French when teaching even though the language of instruction is English as shown below:

Teacher: Everything is in English, they have to try and learn English. We are teaching in English but for mathematics sometimes I switch to French for those who understands little English.
Interviewer: Are there French or Portuguese lessons for these learners?
Teacher: There is none. There are no French lessons but for me I try and explain in French but because but for those from Portuguese-speaking countries they have to learn English. Maths sometimes you can play with language, they understand but if they do not understand I try and explain to them in French. But they have to learn English. Generally they have to speak English.
Interviewer: Do you allow them to present their mathematical knowledge in French?
Teacher: For me I try, but they have to learn English everything is in English.

The unitary approach means that English should be used to teach linear programming to immigrant students and immigrant students have to learn English in order to participate. The use of another language like French was clearly not desirable during the teaching and learning of linear programming.

However, centrifugal forces were evident, in the teacher's use of French during the teaching of linear programming to sensitise immigrant learners who understood French. For example, the teacher introduced the French version of the word aquarium to aid the learners with understanding this word. Heteroglossia is noticeable in his pronunciation of the word in French and the way that he positioned himself as someone who can read English but cannot pronounce it properly in English.

Teacher: A school wants to take learners on an outing, a school wants to take learners on an outing to an aquar-aquarium, eh! How do you pronounce it I don't know in English aquarium or aquarium? Because in French we pronounce it as aquarium [teacher introduces French pronunciation] in an outing to an aquarium

The role of centrifugal forces was clear during lesson observation where learners were constructing inequalities from given statements in English. The teacher reformulated his explanation of some words into French, so that the linear programming content would be meaningful to immigrant learners. This practice led to the development of immigrant learners' knowledge of the mathematical skills needed when solving a linear programming task. The translation of selected words was based on immigrant learners' prior knowledge of French regardless of the fact that it was not the language of instruction in South Africa:

… a workshop is available for 20 days each month … at least 2 units of Ralto must be produced each month …

Teacher: As I cannot go beyond this less or equal to and here they say availability. Aaah! Availability … this is a constrain when they say availability … Availability implies less or equal to et cela signifie moins de (*and that means less than)* therefore will have the constrain here…

Teacher: At least two unit of Ralto must be produce each month, y a au moins deux at least two units of Ralto. Alors … sens … hein plus grand que deux Ralto … This is the first constrain … they say at least two units, at least two units means what?

Learners: Greater than

Teacher: Greater than or equal, at least two units of Ralto … It means x must be at least 2. I cannot produce less than two units for Ralto. Je ne peux produire moins pour que deux Ralto …

The teacher switches between English and French in order to enable immigrant learners to understand the mathematical meaning of 'available' and 'at least' in relation to the linear programming task. This approach may support immigrant learners to learn to communicate in mathematical English. In this multilingual mathematics classroom, the teacher mediates between English and the French the immigrant learners brought to the class. This form of mediation is a result of the centripetal and centrifugal forces present in this specific context.

10.8 Discussion and Conclusions

In each of the four cases described in this chapter, tensions are apparent between prevailing assumptions that reflect a unitary language perspective and the heteroglossia of mathematics classrooms in the context of language diversity. Centripetal forces are felt through language policies, parental pressure, and teachers' or students'

beliefs and preferences. In Malaysia, the national language policy to use English as the language of mathematics instruction; in Québec, a provincial language policy requires immigrants to learn in French; in South Africa, school language policies follow parental preferences for English. There are some interesting contrasts however: in Malaysia, parental pressure led to officially sanctioned diversification of the languages used for mathematics instruction, while in South Africa, the diversity permitted in national policy is reduced by a widespread preference for English. Hence, while language policy tends to act as a centripetal force in mathematics classrooms, the uptake of such policies is shaped by local responses, notably from parents.

Heteroglossia is apparent in all four cases, and includes the use of multiple languages, multiple discourses and variations in accent, word choice and the use of gesture and a range of representation systems. In the Malaysian and South African examples, code-switching was, despite teachers' stated preferences, regularly deployed to enhance students' access to mathematical meaning or the meaning of mathematical problems. In the Québec case, by contrast, there was little evidence of code-switching, despite some of the students sharing a common language. Instead, the students in the Québec case used accented, highly idiosyncratic French, and required substantial mediation on the part of the teacher to interpret their mathematical thinking and guide their expression of their ideas.

It is important to note, however, that heteroglossia is not only about code-switching; for example, several of the cases shown here exemplify the use of a range of levels of formality in mathematical expression. In the first case, the teacher encourages students to consider an everyday context for their problem, and in the third case, the students' efforts to describe convex shapes draw on a variety of informal forms of expression, including gestures and the use of nonspecific forms of reference (e.g., this, those). Again, the teacher plays a key role in mediating between these different levels of formality.

The centripetal and centrifugal forces are clearly in tension in our four cases. The teacher in case 1, for example, says that students should learn in their main languages so that they understand mathematics, but also that they should learn in English, because it is an international language. In case 2, a single language policy is negotiated into a dual language policy, but in practice a mixture of languages is used in mathematics classrooms. In case 3, a student expresses a desire to use Spanish to explain his mathematical thinking but the teacher, the program, and the province all insist on French. And in case 4, English is the medium of instruction, but the teacher mixes it with French. These tensions, moreover, are more complex than they are described here. In three of the cases, not all students share all languages in their mathematics classes and in cases 3 and 4, the only language shared by everyone including the teacher is English.

Teachers' mediation of this tension takes several forms. In some cases, it involves translating, code-switching or using language mixtures to provide additional forms of mathematical meaning making for students. In all cases, it involves policing the language policy, sometimes strictly (e.g., case 3), sometimes not (e.g., case 1). It can also involve the use of multiple discourses (e.g., everyday vs. mathematical, case 1) and systems of meaning (e.g. gesture, case 3). Finally, the teacher plays a central

mediational role in all cases in interpreting and making mathematical meaning with students' utterances, including gestures, such as by re-voicing them using different words, a different language or mathematical symbols.

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