Chapter 5
The Electoral College and Campaign Strategies

Abstract Under the current presidential election system, a set of 51 concurrent elections—in each of the 50 states and in D.C—constitute a presidential election. Campaigning throughout the country requires every presidential candidate to spend financial resources as effectively as possible. Each candidate has limited time to demonstrate to the voters that she/he is the best fit to be President, and the amount of time remaining before Election Day decreases with every passing day. This chapter focuses on how the Electoral College affects campaign strategies of presidential candidates, and how this election mechanism helps evaluate strategic and tactical abilities of the candidates. The chapter provides verbal formulations of problems to be solved by the teams of presidential candidates in planning election campaigns. It demonstrates the analogy of these problems to pattern problems solved in transportation systems. This analogy allows one to use well-developed software for solving both mathematical programming and discrete optimization problems in planning and analyzing election campaigns. The chapter discusses two extreme election strategies aimed at throwing the election into Congress in an attempt of a presidential candidate to win the Presidency there, by bypassing the Electoral College.

Keywords Allocating financial and time resources · Bin-packing · Campaign strategies · Combinatorial problems · Extreme election strategies · Feasibility test · Knapsack problem · Misleading campaigns · Probabilistic estimates · Routing · Schedules of visits to the states · Strategic and tactical abilities of a presidential candidate

Under the current presidential election system, a set of 51 concurrent elections—in each of the 50 states and in D.C—constitute a presidential election. Campaigning throughout the country requires every candidate to spend financial resources as effectively as possible. Each candidate has limited time to demonstrate to the voters that she/he is the best fit to be President, and the amount of time remaining before Election Day decreases with every passing day.

A.S. Belenky, *Who Will Be the Next President?*,
DOI 10.1007/978-3-319-44696-7_5

The country usually judges how effective the campaign of a particular presidential candidate is based upon the nationwide polls that are conducted by reputable polling organizations. Even if correctly conducted and processed, generally, these polls cannot project the victory of a particular presidential candidate in the Electoral College [1].

Nevertheless, they allow one to conclude whether the candidate is gaining or losing voter support, which says something about how successfully or unsuccessfully her/his campaign is being run. This, in turn, allows one to draw certain conclusions on the ability of the candidate to select people for the team and to lead, which reflects the candidate's strategic abilities.

Certainly, each presidential candidate who has a chance to win in the Electoral College has a campaign strategy that is not revealed and should not be revealed to the public. Yet every interested voter has a chance to decide which states are key for a particular presidential candidate and to follow campaign developments there on a day-by-day basis proceeding from all the available information communicated by the media and by the Internet. The analysis of this information, as well as that of the messages delivered by both the candidate and her/his competitors in the race, can give the voter additional information on the strategic abilities of the candidate.

This chapter focuses on how the Electoral College affects the campaign strategies of presidential candidates, and how this election mechanism helps evaluate the strategic and tactical abilities of the candidates. Also, it provides verbal formulations of problems to be solved by the teams of presidential candidates in planning election campaigns and demonstrates the analogy of these problems to pattern ones solved in transportation systems. This analogy allows one to use well-developed software for solving mathematical programming and discrete optimization problems [1] in planning and analyzing election campaigns. The chapter briefly discusses two extreme election strategies aimed at throwing the election into Congress in an attempt by a presidential candidate to win the Presidency there, by bypassing the Electoral College.

5.1 The Electoral College and the Logic of Winning the Presidency

Since the states appoint their presidential electors based upon election results there, rather than on any national tally, a presidential election is a set of statewide elections in 50 states and a district-wide election in D.C., which are run concurrently. Constitutionally, in summing up the results of these concurrent elections, the Electoral College can act as an independent body. However, since the adoption of the "winner-take-all" method for appointing state electors by a majority of the states, most of the time, it has acted as a "rubber-stamp" body. It mirrors the state popular vote results, i.e., the numbers of electoral votes won by the candidates in the statewide elections and in a district-wide election in D.C.

Under the current election rules, all the competing presidential candidates with a chance of winning the election in the Electoral College try to allocate their financial and time resources to win as many electoral votes as they can. Thus, each candidate should be concerned with two problems: (a) how to estimate her/his chances to win the election in the Electoral College, and (b) how to allocate available financial and time resources to win the election there (if these chances are real).

Solving the first problem involves certain probabilistic and combinatorial calculations, since estimating the chances of any event to occur means calculating the probability of this event to occur. Solving the second problem implies finding available variants of winning a majority of all the electoral votes that are in play in the election (currently, 270 if all the states and D.C. appoint all the electors that they are entitled to appoint), and choosing some variants from among the available ones is a combinatorial problem.

Contemporary mathematics offers powerful tools for solving both problems, and a brief description of the ideas underlying these tools and their potential for solving both problems are the subject of the discussion to follow.

One can divide all the 50 states and D.C. into three sets of places awarding electoral votes, and the team of a presidential candidate should make a division of the places both for the candidate and for all of her/his competitors in the race who have real chances to win the Presidency in the Electoral College. The first set, A1, is formed by the places in each of which the candidate can be sure to win all the electoral votes. The second set, A2, is formed by the places in each of which the candidate has no chance of winning the electoral votes, and her/his competitors consider the places from this set to belong to their sets A1. The third set, A3, is formed by the places in each of which the candidate can eventually win by competing with the opponents; these places are often called "toss-up" ones [1, 48].

Certainly, such a division of 50 states and D.C. into the above three sets of places should be considered by the team of each particular candidate. This division depends on both the candidate's team and the current status of the race. In the course of the election campaign, places from one of the three sets at one stage of the campaign can be reassigned to one of the other two sets at the next stage. However, whatever the current division, the candidate's team needs to find the best allocation of money and time available to the candidate at any stage of the campaign through the end of the race.

Let us assume that 538 electoral votes are in play in a presidential election. It is natural to assume that if the set A1 for a particular presidential candidate consists of the places that govern N1 electoral votes combined, this number does not exceed 269. If the set A2 consists of the places that govern N2 electoral votes combined, this number also does not exceed 269. Let the set A3 consist of the places that control N3 electoral votes combined. Then N3 should be such that at least one of the participating candidates would be able to win as many electoral votes in the places from the set A3 as she/he needs to win at least 270 electoral votes combined.

How should the candidate's team allocate the available resources of both kinds (i.e., money and time)? Allocating the resources in places from the set A3 should concern the team the most, while sufficient attention should be paid to places from the set A1 to preserve their loyalty on Election Day. Also, the team may decide that campaigning in some places from the set A2 may make sense for the candidate if this may affect the campaign of her/his major opponent in the race who may consider these places to belong to her/his set A1. Such a move of the candidate may force the opponent to spend more resources in these places than the opponent would have spent otherwise and weaken her/his chances to win in places from the set A3 in which the candidate needs to win electoral votes.

Example 5.1 [1, 48]. Let the set A1 for a presidential candidate in the 2016 election consist of the following 18 places:

1) California (55)	7) New Jersey (14)	13) D.C. (3)
2) New York (29)	8) Massachusetts (11)	14) Alabama (9)
3) Florida (29)	9) Tennessee (11)	15) Connecticut (7)
4) Vermont (3)	10) Hawaii (4)	16) Delaware (3)
5) Rhode Island (4)	11) New Hampshire (4)	17) West Virginia (5)
6) Alaska (3)	12) Maryland (10)	18) Arkansas (6)

Let the candidate's team believe that the candidate cannot win electoral votes in the following places forming the set A2:

19) Texas (38)	25) Michigan (16)	31) Indiana (11)
20) Pennsylvania (20)	26) North Carolina (15)	32) Wisconsin (10)
21) Illinois (20)	27) Georgia (16)	33) Missouri (10)
22) Ohio (18)	28) Virginia (13)	34) Washington (12)
23)Maine (4)	29) Nebraska (5)	35) Kansas (6)
24) Oregon (7)	30) Utah (6)	36) New Mexico (5)

Then the set A3 for the candidate consists of the following places:

37) Arizona (11)	42) South Carolina (9)	47) Idaho (4)
38) Minnesota (10)	43) Oklahoma (7)	48) Montana (3)
39) Colorado (9)	44) Iowa (6)	49) Wyoming (3)
40) Louisiana (8)	45) Mississippi (6)	50) North Dakota (3)
41) Kentucky (8)	46) Nevada (6)	51) South Dakota (3)

For the sake of certainty, the places from the sets A1, A2, and A3 are listed along with the numbers of the electoral votes that these places control in the 2016 election. To win the election, the candidate needs to win at least 60 electoral votes from the set A3.

The Electoral College rules and the tradition of presidential election campaigns dictate some general principles of allocating both financial and time resources that the teams of the candidates should bear in mind. While most of the candidate's attention should go to the places from the set A3, each candidate's team should spend a certain amount of money to continuously air advertising messages both in the set A1 and nationwide. These messages should target particular categories of eligible voters such as women, youth, retirees, middle class voters, etc. Most of the remaining money should be spent in places from the set A3, which are "toss-up" ("battleground") ones. For each place from the set A3 the candidate's team should estimate how much money and time is sufficient to spend to win the electoral votes there.

When the candidate campaigns in some places from the set A2 in an attempt to force her/his major opponent in the race to divert parts of the opponent's resources from spending in places from the set A3, she/he runs a tactically misleading campaign. However, tactically misleading campaigns may or may not be effective. Moreover, the opponent may apply the same tactic against the candidate, by intensively campaigning in places from the candidate's set A1. Therefore, the candidate's team should decide how much money and time to spend in places from the set A2 while holding on to her/his chances in the places forming the set A1.

No matter whether the candidate's team uses expert estimates or calculates the amount of each of the two resources to be spent campaigning in each place, the campaign develops in a competitive environment. This means that all the estimates can be considered true only with certain probabilities, which depend on such factors as, for instance, the economic situation in the country, the international political climate, and the campaign strategies of the candidate's major opponents. These probabilities are estimated using probability theory. Elementary concepts and facts of this theory, along with examples illustrative of using these concepts and facts in the context of U.S. presidential elections, are presented in the author's books [1, 48].

How can the candidate's team calculate the estimates of how much money is needed to win at least a plurality of votes for the candidate's electors in a state or in D.C.?

There are similarities between advertising goods and services that a company tries to sell to targeted customers and advertising programs, promises, and personal qualities of the candidate that her/his team tries to "sell" to the voters in the course of an election campaign [1, 49]. In fact, the candidate's team should plan an advertising campaign aimed at "selling the features" that the candidate possesses in an attempt to convince eligible voters to support the candidate by favoring her/his electors in their respective places. These similarities allow the candidate's team to use well-developed approaches to planning advertising campaigns of goods and services. Mathematical methods implementing these approaches, particularly, those presented in [50, 51], allow the candidate's team to estimate the amounts of money needed to win in each particular place from the sets A1 and A3.

To plan the campaign, the team needs these estimates (calculated or obtained from the experts) and those of the available amount of money that the candidate's team can afford to spend for campaigning in places from the set A3. Proceeding from this data, the team can find a combination of the places (or a set of combinations of the places) where the candidate should focus her/his campaign. Here, at every particular time in the course of the election campaign, the available amount of money to spend in places from the set A3 is the difference between the amount that the candidate has raised and the amounts that will be spent (a) for nationwide advertisements, (b) for campaigning in places from the set A1, and, (c), possibly, for campaigning in some places from the set A2 (if conducting a tactically misleading campaign is part of the candidate's campaign strategy).

The candidate's team is interested in finding so-called "victorious" combinations of places from the set A3 [1, 48]. These places are those where the candidate should campaign in hope to win the Presidency in the Electoral College. (To this end, currently, at least 270 electoral votes are to be won in the chosen places from the set A3 and in the places from the set A1 in which the candidate is "guaranteed" to win.) Any "victorious" combination of the places from the set A3 in which the candidate can win is associated with an amount of money to be spent there. Certainly, the candidate's team should be interested in finding "victorious" combinations of the places requiring the minimum financial expenditures for successfully campaigning there while securing an acceptable reliability level of the results expected from campaignning in thus chosen places. Once such a particular "victorious" combination of the places (i.e., a combination of the places requiring the minimum financial expenditures) has been selected, the candidate's team should find the best routes of visiting the places from the sets A1, A3, and, possibly A2. Well-developed methods for solving routing problems can be used to this end [52]. It is clear that to remain competitive at all the stages of the campaign, the candidate's team may need to do all these calculations many times as the election campaign develops.

5.2 Allocating Financial and Time Resources

Let the minimum amount of money to win at least 270 electoral votes combined be calculated at a particular stage of the election campaign. Then the candidate's team should compare the amount of money that is available to the team at this stage and this calculated minimum amount. If the available amount is smaller than the minimum one, additional money should be raised. This may require the candidate to make additional visits to some "donors," in particular, to those from the states comprising the set A1. Also, in calculating the routes of visits to places from the sets A1 and A3, one should take into account certain obligations that the candidate may have by the time of the calculations. All this, along with the reliability reasons, requires raising more money than the minimum amount needed for campaigning in the chosen "victorious" combination of places from the set A3. Usually, the money that is needed for campaigning is considered a more precious resource than the time remaining before Election Day.

To illustrate how the problem of finding a “victorious” combination of places may look in the simplest case, let us consider an example from [1] (though with the data corresponding to the 2016 election).

Example 5.2 [1]. Let A1, A2, and A3 be the same sets of states as those from Example 5.1 for a presidential candidate in a campaign.

Let the candidate have $100 million left for campaigning, and let 90 days remain before Election Day. Further, let the candidate need to spend $49 million and 60 days in places from the set A1 (a) to preserve the loyalty of the candidate’s supporters there on Election Day, and (b) for conducting nationwide campaign activities and airing nationwide advertisements. This means that the candidate can afford to spend $51 million for campaigning in states from the set A3, and that only 30 days are left before Election Day for campaigning there.

Finally, let the candidate need to spend the following amounts of money and time for campaigning in each of the states from the set A3 to win the electoral votes there:

37) Arizona	11 electoral votes	8 million	5 days
38) Minnesota	10 electoral votes	7 million	5 days
39) Colorado	9 electoral votes	8 million	5 days
40) Louisiana	8 electoral votes	6 million	5 days
41) Kentucky	8 electoral votes	8 million	5 days
42) South Carolina	9 electoral votes	7 million	4 days
43) Oklahoma	7 electoral votes	8 million	5 days
44) Iowa	6 electoral votes	7 million	4 days
45) Mississippi	6 electoral votes	7 million	3 days
46) Nevada	6 electoral votes	4 million	2 days
47) Idaho	4 electoral votes	5 million	3 days
48) Montana	3 electoral votes	3 million	2 days
49) Wyoming	3 electoral votes	4 million	3 days
50) North Dakota	3 electoral votes	4 million	2 days
51) South Dakota	3 electoral votes	3 million	3 days

First, the candidate’s team should find whether the available money and time resources are sufficient to let the (electors of the) candidate win at least 60 electoral votes in states from the set A3. If they are sufficient, at least one “victorious” combination of states from the set A3 can be found. The candidate can win the election by winning all the electoral votes (at least 60) in this “victorious” combination of the states, along with 210 electoral votes in the states from the set A1. Second, if there are more than one “victorious” combination, the candidate’s team can choose the one that best meets some other requirements that the candidate may need to meet. If the available amounts of both money and time resources do not let the candidate win at least 60 electoral votes combined, the candidate’s team should find whether there is enough money available to win at least 60 electoral votes in states from the set A3 (no matter whether the time limitations hold). If there is, the candidate's team may try to "compress" the schedule of the candidate's visits to the states in the remaining part of the campaign.

Finally, if the available amount of money is not sufficient for winning at least 60 electoral votes in places from the set A3, the candidate's team should determine potential "donors." These "donors" can be from the set of states A1, where the funds needed for campaigning in states from the set A3 can be raised. The team should recalculate the schedule of visits of the candidate for the remaining part of the campaign. Once the amount of money needed to campaign in states from the "victorious" combination of states from the sets A1 and A3 has been raised, the team should recalculate the allocation of all the available resources.

Proceeding from the data for the 15 states forming the set A3, one can be certain that, for instance, a combination of the following 7 states is "victorious" for the candidate, since these states govern 60 electoral votes combined. Moreover, the available amount of money ($51 million) allows the candidate to campaign and succeed in winning at least 60 electoral votes in states from the set A3. However, the number of days required for successfully campaigning in these particular 7 states equals 31, exceeding the available number of days (30 days) by one day.

37) Arizona	11 electoral votes	8 million	5 days
38) Minnesota	10 electoral votes	7 million	5 days
39) Colorado	9 electoral votes	8 million	5 days
40) Louisiana	8 electoral votes	6 million	5 days
42) South Carolina	9 electoral votes	7 million	4 days
43) Oklahoma	7 electoral votes	8 million	5 days
46) Nevada	6 electoral votes	4 million	2 days

The time required for campaigning in each place includes that for (a) transportation to and from the place, (b) accommodation in the place, and (c) rest. The required amount of time much depends on the candidate's physical ability to withstand a "compressed" schedule at a particular stage of the campaign. In the illustrative example, the candidate's team can suggest several options to "compress" the candidate's schedule of visits to the states from the "victorious" combination of the states from the set A3.

In general, time is usually considered a more flexible parameter of the campaign than the money needed for successfully campaigning (see Sect. 5.1). Such an approach leads to solving mathematical problems that are simpler than those in which both money and time are treated as equally important parameters.

The author's publications [1, 48, 49] consider mathematical formulations of problems associated with planning campaigns of presidential candidates. These problems include (a) those of verifying whether the available amount of money is sufficient for successfully campaigning in places from the sets A1 and A3, and

(b) those of finding an additional minimal amount of money to be raised if need be. These problems cover the case of treating money and time as equally important resources, as well as the case with the money being a less flexible and the time being a more flexible resource. References to software available for solving these problems in both cases can be found in [1, 49].

Mathematical models proposed in [1, 48, 49] enable the candidate's team to find an optimal allocation of both resources for campaigning in places from the set A3 only if the available amount of money is sufficient for successfully campaigning there (i.e. for winning at least 60 electoral votes in the case considered in Example 5.2). This money is to be spent for campaigning in each of the places from any "victorious" combination of places from the set A3. If the available amount is not sufficient, the models help determine the minimal additional amount of money to raise. Also, these models can help the team determine the allocation of the increased amount of money among "victorious" combinations of places from the set A3.

The teams of all the candidates with a chance of winning in the Electoral College are likely to calculate an optimal allocation of both resources at different stages of the campaign. Indeed, the set A3 may change several times in the course of the campaign, and this set of places should control enough electoral votes combined to let the candidate win in the Electoral College by winning in places from the sets A1 and A3. Therefore, tools for effectively allocating both resources are needed.

It turns out that the problem of allocating financial resources is completely analogous to a well-known discrete optimization problem [52]. Consider a person who is going to spend at least 270 days on an island and wants to eat homemade food. This food is available in 51 packs, and different packs contain different food. The food in each pack is sufficient to feed the person for a particular number of days. This number is different for different packs and falls within the range of 3–55 days.

Each pack has the weight and the volume known to the traveler who plans to put the packs in a knapsack. The knapsack can accommodate a set of packs provided the total volume of them does not exceed the volume capacity of the knapsack. The traveler can carry a weight that does not exceed her/his physical ability.

The analogy between the candidate's problem and that of the traveler becomes obvious if one notices that [1, 48]

(a) the knapsack volume can be viewed as an analog to the amount of time available for campaigning until Election Day, which must not be exceeded;
(b) the traveler's physical ability to carry a weight can be viewed as an analog to the amount of money that is available to the candidate until Election Day, which also must not be exceeded;
(c) each pack with a particular food from among 51 packs can be viewed as an analog to a place awarding electoral votes (state or D.C.);
(d) the weight of each pack can be viewed as an analog to the amount of money that the candidate should spend for campaigning in the corresponding place to help her/his electors win;

(e) the volume of each pack can be viewed as an analog to the amount of time that the candidate should spend for campaigning in the corresponding place,

(f) the number of days that the food from a pack allows the traveler to eat normally can be viewed as an analog to the number of the electoral votes that the corresponding place governs, and

(g) the at least 270-day duration of the journey can be viewed as an analog to the number of the electoral votes that the candidate expects to win in the election.

In deciding whether to undertake the trip, the traveler tries to estimate (a) which packs to put in the knapsack to eat normally for at least 270 days, and (b) how much the loaded knapsack will weigh. In deciding how to run the election campaign, the candidate's team tries to find which states should support the candidate to secure her/his victory. This means that her/his team should decide which states form the sets A1 and A3, and in which states from the sets A1, A3, and, possibly, A2 to run the campaign. (The traveler can certainly consider that some mandatory packs are to be put in the knapsack—and these packs are analogous to places forming the set A1—and to choose places from the sets A3 and A2 only.) Thus, both the traveler and the presidential candidate's team face the same mathematical problem: how to find the best composition of items of known volumes and weights (for the traveler) and that of known money expenditures and time (for the candidate) to put in a knapsack of a known volume and a known weight (expenditures and time for the candidate) for the maximum effect.

Mathematically, this problem is a particular case of a bin-packing problem—a two-dimensional Boolean knapsack problem with an additional constraint [1]. Bin-packing problems are well studied in applied mathematics, and various mathematical methods are known for their solution [53].

5.3 Optimizing the Candidate's Schedule

Let a subset of places from the set A3 and a subset of places from the set A1 be chosen by the candidate's team for campaigning and fundraising at a particular stage of the election campaign. Then the team should choose a sequence of visits to these places that the candidate should follow. That is, a set of routes connecting the chosen places, each to be visited a certain number of times, should be developed. Each visit may include different activities, and usually includes a set of meetings at town halls, at universities, schools, etc. to attend and a set of appearances on TV and radio stations to make in several cities within a place (state or D.C.). All kinds of transportation means— airplanes, trains, buses, river boats, etc.— can be used by the candidates to travel each route, and schedules of the candidate's competitors who may decide to visit the same place at the same time should be taken into consideration.

As before, an analogy between the problem that the candidate's team faces in developing an optimal set of routes and the problem that, say, a truck driver faces in choosing an optimal scheme for delivering beer to a set of recipients located in a set of places is obvious. It is this analogy that allows the candidate's team to use a theory of routing [52] to build an optimal schedule of visiting those places from the set A3 that, together with places from the set A1, form the chosen "victorious" combination of the places to campaign, as well as to do fundraising.

There are several routing problems that have been studied by mathematicians, called pattern routing problems, for which solution algorithms and software have been developed [52]. In one of these problems, called the traveling salesman problem, a transportation means (a truck) starts its route at a particular place (called the base), makes visits to several customers to deliver them commodities (bear), and returns to the base. Proceeding from the cargo-carrying capacity that the truck has and from the geography of the customer locations, one should find an optimal sequence of visits. That is, one needs to find an optimal route of the truck, taking into account the distances or the time needed to travel between each pair of the customer locations and between each customer location and the base. The optimality is understood in the sense of the total time that the truck needs to travel to visit every customer [52].

One can view the cargo-carrying capacity of the above transportation means (truck) as the amount of time that the candidate has left before Election Day to visit the places from the sets A1 and A3 to campaign and to raise money there. The places can be viewed as the customers to visit, the base can be viewed as the headquarters of the candidate's campaign, and the time to travel between the base and each place, as well as the time to travel between each pair of the places to visit, and, possibly, to spend in particular places, is known. This makes the routing problem for the candidate's team to solve completely identical to that to be solved by the truck driver (travelling salesman) for the truck (transportation means).

As a matter of practice, during one visit, the candidate will never visit all the places that she/he needs to visit from any "victorious" combination of places from the sets A1 and A3 (calculated at any stage of the election campaign). The candidate is likely to visit several places and then to return to the headquarters, to spend some time there, and only then to continue visiting the remaining places. Also, the candidate must not necessarily return to the headquarters after every tour, and the candidate may have more than one headquarters or may have several places in which the tours may both start and end.

The travelling salesman problem can serve as an example of a routing problem that the candidate's team may need to solve at some stages of the election campaign. There are other problems that the team may consider that have the same kind of analogy with the problems that are solved for a transportation means. For instance, if the team would like to partition a set of all the places to visit into several closed tours, i.e., to partition all the places from the sets A1 and A3 to be visited into a few subsets, another well-known routing problem can be used to find the optimal partitioning.

The "p-travelling salesmen problem" allows one to find this optimal partitioning into p subsets of the set of the places from A1 and A3 in such a manner that (a) each subset of the places is to be visited by the candidate on a separate tour, (b) each place from the sets A1 and A3 is visited only once and only within one of the tours, and (c) the candidate visits all the places and returns to the headquarters after each tour [52].

If the candidate's team decides that the candidate should have several headquarters or several places from which he can start several tours, the so-called "*p* travelling salesmen problem with *p* bases" can be used for developing the candidate's set of the tours. Here, the routes should not necessarily be closed, and the candidate may start the next route at the place where the previous route ended. The interested reader may find many other examples of the pattern routing problems in the author's book [52].

Though the verbal formulations of the pattern routing problems may look quite simple, these problems form a class of mathematical problems that are the most difficult to solve from the computational viewpoint [52, 53]. Also, the pattern routing problems may not fit all the needs of the candidate's team in calculating the set of her/his routes at any or at a particular stage of the election campaign. Thus, the candidate's team may need to formulate more complicated problems, tailored to meet more requirements of the tours of the candidate than the pattern routing problems can meet in principle.

Whatever the formulations of the routing problems to be solved, there is standard software, as well as some experimental one, that can be used. This software can be used either directly or become a part of a decision-support system that the candidate's team may need to calculate and recalculate the routes many times in the course of the election campaign. The ability to be adaptive and flexible in adding particular requirements of the candidate's team via a friendly interface is one of the important features of such a decision-support system. Having this system at the candidate team's disposal can make a difference in choosing the best strategies to win. Digitalized geographical data relating to the location of the places from the sets A1 and A3 to be visited by the candidate is now widely available on the Internet, as well as from commercial sources.

5.4 Applying Mathematics to Win

How should the planning of the election campaign of a candidate be organized? If the candidate plans to win the election via the Electoral College, the campaign should include the following:

1. *Strategic planning*. Members of the candidate's team responsible for strategic planning of the campaign, along with the experts, should determine which of the 50 states and D.C. should be assigned to the sets A1, A2, and A3. Then they should determine the set of topics to be addressed by the candidate in the course of visiting these places, a set of advertising messages to be spread both there and nationwide, and the schedule of the appearance of all these messages. Further, they should provide the estimates of time and money needed to campaign in the

places from the sets A1 and A3, including the expenses associated with the advertising, to have a chance of winning the electoral votes there. After that, the amount of money and time for campaigning nationwide should be estimated.

To this end, mathematical problems similar to those in planning advertising campaigns of goods and services, formulated in [50, 51], should be solved.

2. *The feasibility test*. The candidate's team should determine whether the available amount of money is sufficient to successfully campaign in the places from the sets A1 and A3 that control at least 270 electoral votes combined. That is, the team should determine whether there is a chance of winning all the electoral votes there. To this end, problems of the bin-packing kind, mentioned in Sect. 5.2, should be formulated and solved. (In reality, the problems to be solved for running the feasibility test are more complicated than the problems verbally formulated in Sect. 5.2.)

If the money is sufficient, a "victorious" combination of the places from the set A3 should be chosen from solutions to these problems. If it is not, the candidate's team should (a) calculate the minimum amount of money that needs to be raised to have a chance to win the Presidency in the Electoral College, and (b) indentify "places-donors" to visit to raise money, along with the estimates of the time needed to visit them. Then the team should solve the same problems of the bin-packing kind proceeding from the amount of money enlarged by the amount expected to be raised (which is to be not smaller than the calculated minimum amount).

The interested reader can find the mathematical formulations of all the problems that are to be solved to run the feasibility test in [1, 48, 49].

3. *Developing the sequence of visits to the places*. Once the time and money have been allocated among the places from the sets A1 and A3, a sequence of visits to these places, as well as to the "places-donors" should be developed.

To this end, routing problems, reflecting the peculiarities of the visit to each place to be visited, should be formulated and solved. The reader interested in seeing mathematical formulations of the routing problems that may cover the needs of a particular presidential candidate, as well as the description of ideas underlying methods for solving these problems, can find both in [52].

4. *Structuring campaigns in the places from the set A3*. A number of visits to a particular place from the set A3 depends on how effectively the candidate competes there. The structure of allocating the money available for campaigning in a state or in D.C. determines the strategy of the election campaign there. The allocation of the money should be done among all the activities that the candidate plans to conduct in the place.

Examples of mathematical problems to be solved to find, for instance, the best allocation of available financial resources among media markets, printed advertising messages, and any other possible forms of advertisements and campaign activities can be found in [1, 48, 49].

5. *Updating campaign strategies.* All the candidates run their campaigns in a competitive environment. This means that whatever move a particular candidate makes, this move produces countermoves from her/his opponents. These countermoves may necessitate substantial updates or even major changes in the campaign strategy of the candidate. For instance, if one of the candidate's major opponents increases campaigning activities in a state from the set A3, this may force the candidate to spend more money and time for campaigning in this state. If this is the case, this change may trigger the recalculation of the remaining part of the candidate's campaign, including that of the amounts of both the time and money resources needed and the current sequence of visits to the places.

To decide whether to make changes in the candidate's campaign to neutralize any impact that the countermoves of the candidate's opponents may have, the candidate needs a decision-support system. Any decision on making changes should be adopted based upon certain criteria that the candidate's team should have. The system should allow the team to verify whether the changes are needed as many times as the situation may require. The core of the system should consist of mathematical tools to solve the above-mentioned problems. Some of these tools in the form of mathematical models formalizing the problems are described in the author's books [1] and [48]. Methods for solving problems formulated with the use of these models are widely available and are implemented within commercial and open-source software packages [48, 49].

Developing a special decision-support system or appropriately customizing any already developed systems will help the candidate's team calculate competitive campaign strategies. This will give the candidate an advantage over any of her/his competitors who do not use such systems.

As usual, the mathematical analysis of the management strategies and moves of the competitors are likely to secure a competitive edge for those who use this tool. However, as in advertising any goods or services, in advertising the qualities of the candidate, the subject of the advertisement matters. In elections, the ability to deliver the message to the voters and to convince them that the candidate's program is better and her/his character is stronger than those of her/his opponents matters a great deal. If this important ingredient of the candidate's campaign is present, the use of mathematics can turn even a small advantage into a landslide victory. Otherwise, though the use of applied mathematics may help improve the election result a lot, it may not be sufficient to win.

Another problem that the candidate may face is her/his team's opposition to using any sophisticated tools, especially if the advice the tools give contradicts the intuition of her/his team members with respect to their understanding of the voters' mood. If this is the case, there is the chance that the campaign may be in danger. If in the end, the candidate relies on the advice of her/his close friends only, she/he may lose strategically to an opponent who uses mathematics in making decisions. Any unexpected advice that mathematics may give should alert both the candidate and her/his team rather than being ignored or rejected. Indeed, it may signal something invisible to or unexpected by the campaign strategists.

Applied mathematicians should learn from campaign managers about strategic principles that the candidate adheres to. In turn, campaign managers should ask applied mathematicians to help detect covert strategic moves of the candidate's opponents using mathematical tools. Only such a cooperation may help the team avoid irreversible losing situations in the campaign.

5.5 Gaming the Electoral College

When the Founding Fathers created the Electoral College, they (apparently) did not expect this election mechanism to always determine the next President (see Sect. 1.3). Therefore, they authorized the House of Representatives to make the ultimate decision on the election outcome should the Electoral College fail to elect a President. But this design of the election system created a legitimate way to bypass the Electoral College and to attempt to win the Presidency directly in the House of Representatives.

In fact, the Founding Fathers created two election mechanisms for winning the Presidency—the basic one (in the Electoral College) and the reserve one (in the House of Representatives). However, there is nothing in the Constitution that would prohibit a particular presidential candidate to use the reserve mechanism without using the basic one first.

This situation is similar to the one in which a parachutist who jumps from a plane and has two parachutes—a basic one and a reserve one—may decide to use the reserve one without trying the basic one if (for whatever reasons) she/he doubts that the basic parachute is reliable.

Thus, the natural course of a presidential election that society expects is that all the candidates compete to win the Presidency via the Electoral College. However, one should not rule out that an extreme strategy of throwing the election into Congress may become a strategy for a particular candidate. Moreover, this extreme strategy may be competitive and even the only winning one for this candidate if (a) she/he does not have a chance to win the Presidency in the Electoral College, (b) the party that the candidate represents is expected to control at least 26 delegations in the House of Representatives, and (c) the candidate and her/his party can secure the quorum (of at least two-thirds of 50 state delegations) to start electing a President in the House of Representatives.

To throw the election of a President into the House of Representatives, the interested candidate should (at least) manage to be among the electoral vote-getters with the top three highest numbers of electoral votes received to eventually have a chance to be considered by the House of Representatives in electing President there. Here, she/he may not win electoral votes at all, since a presidential elector may favor her/him for any reason, including a political agreement between the candidate and her/his competitor (whom this elector is expected to favor in the Electoral College). Yet this may be possible only if two fuzzy presidential election rules are interpreted as follows [1, 48]:

Rule 1 A presidential elector as a free agent can favor whomever she/he wants, despite any obligation to favor a pair of particular presidential and vice-presidential candidates and despite any restrictions that some (currently 29) states and D.C. impose on presidential electors. Thus, an electoral vote that has been won by a pair of presidential and vice-presidential candidates, say from party A, may be received by another pair of presidential and vice-presidential candidates or by either person from this pair. These pairs of the candidates (or persons) may be different from those who head the slate of electors to which the above elector belongs.

Rule 2 Congress can always decide how many persons who are recipients of the electoral votes as President should be considered by the House of Representatives in electing a President there. Indeed, the phrase from the Twelfth Amendment "… not exceeding three on the list of those voted for as President …" [19] does not make it clear how many persons from among at least three electoral vote recipients are to be considered in electing a President there, and how to select them.

Thus, besides the option to merge the electoral votes won by several candidates to receive a majority of all the electoral votes in the Electoral College (currently, at least 270), Rule 1 may put a candidate among the top three electoral vote-getters. Therefore, there are three constitutionally allowable election strategies that a candidate may exercise to win the Presidency:

(a) to win a majority of all the electoral votes that are in play in the election,
(b) to merge the electoral votes won by different candidates to accumulate a majority of the electoral votes and to receive this majority in the Electoral College, and
(c) to throw an election of a President into Congress, to manage to become one of the top three electoral vote-getters, and to secure both support from a majority of the state delegations in the House of Representatives and a quorum to start the election procedure there.

While election strategies (b) and (c) are certainly extreme, neither is constitutionally prohibited. Moreover, either may be competitive and even winning in a particular election. Which of these three strategies to exercise depends on which strategy gives the candidate a better chance to win the Presidency. The reader interested in learning how the chances of winning the Presidency in the House of Representatives can be evaluated, is referred to the author's publications [48, 49].

In exercising extreme election strategy (c), the interested candidate should manage not to let any candidate win the election in the Electoral College. It turns out that the "winner-take-all" method for awarding state electoral votes can be exploited to this end. Indeed, the "winner-take-all" method can help "balance" the number of electoral votes that potential Electoral College winners may win in every closely contested state.

If the electors of candidate A are likely to win in a state, the interested candidate may "sponsor" the campaign of the candidate A's major opponent or opponents by arranging debates on some election issues in this state either with the participation of candidate A or even without her/him. The debates should convince a part of candidate A's supporters and independents to support the electors of a candidate A's opponent and not to let candidate A win electoral votes in the state.

The "winner-take-all" method can be exploited the same way by an interested candidate who tries to win the election in the Electoral College by arranging debates in a state from her/his set A2 with the opponents of the race favorite there or with the state's favorite herself/himself. This move may lower the threshold of a plurality of votes needed to win the electoral votes in the state and may let the interested candidate eliminate the chances of the electors of the state's favorite to win electoral votes there. Certainly, such an activity requires the interested candidate to spend money and time in the states in which it will be conducted and to make promises that may interest supporters of the opponents of the state's favorite.

Also, in exercising strategy (c), interested candidate B may need to convince other candidates who have won electoral votes to trade them for her/his promises and to instruct their electors to favor candidate B in the Electoral College [1, 4]. Precedents of elections in which the electors of one presidential candidate favored another presidential candidate are well known, and it was widely expected that such a trade of electoral votes would take place in the 1968 election [4].

5.6 Misleading the Opponents

Conducting a misleading campaign is a powerful tactical weapon that a presidential candidate may deploy, especially in close elections. Under the current election system, a misleading campaign is a set of activities aimed at convincing the major opponent (opponents) (a) to allocate more resources to some of the states from her/his (their) set (sets) A1, and (b) to reallocate her/his (their) resources to the states in which the electors of the opponent (opponents) cannot win by creating in her/him (them) the impression that they can [54]. With respect to part (b) of these activities, the most "reliable" way to create this impression is to affect the polls in the "battleground" states that reflect the state's support for each competing candidate, including that from particular voter groups there [48].

Let us assume that one can artificially affect the poll results in favor of the candidate's closest opponent in a particular state that this opponent considers to be from the set A3 (for this opponent), whereas (at least from the candidate's viewpoint) this opponent does not have a real chance to win there. Then the opponent may decide to switch her/his attention and resources to this state and thus may weaken her/his positions in at least one of the other "battleground" states, helping the candidate win there. Technically, such an effect can be "achieved" by using pollsters who (for a certain period of time) may conduct polls on the samples of state voters that disproportionally include people favoring the candidate's opponent. Conducting these misleading polls and announcing their results may be coupled with announcing the intent of the candidate to switch her/his campaign to other "battleground" states and explaining such a move by (allegedly) decreasing her/his chances to win electoral votes in the state.

If the candidate's opponent does not recognize the misleading nature of the candidate's move, she/he may make a fatal mistake by switching more of the remaining resources to the state to which she/he would have never switched them, otherwise. By doing so, the opponent is likely to take these resources from other "battleground" states, and if this is done close to Election Day, the consequences of such a decision may be irreversible [48, 54].

While this strategy is certainly extreme and (if exercised intentionally) can be considered a form of manipulation of public opinion, the candidate may exercise a different though a similar one corresponding to the above set of activities (b). That is, the candidate may announce the intent to win in a state from the set that the candidate's opponent considers to be from her/his set A1, i.e., in a state loyal to the opponent. This strategy may enforce similar changes in the opponent's plans for the remainder of the election campaign.

The power of creating a wrong impression in the opponent's mind was demonstrated in the course of the 2000 and the 2004 election campaigns in which strategic mistakes made by the teams of the candidates who ran poll-driven campaigns caused these candidates defeat in both elections.

Conducting misleading campaigns in a state or in a set of states may backfire. Reporters, political observers, and TV and radio talk show hosts may eventually be deceived by the candidate's move, which may negatively affect the candidate's real chances in this state or in these states. Also, making misleading moves requires extremely thorough calculations and the use of sophisticated mathematical methods for all the probabilistic estimations.